MATHEMATICS 8

Three-Dimensional Geometry

Distance Learning

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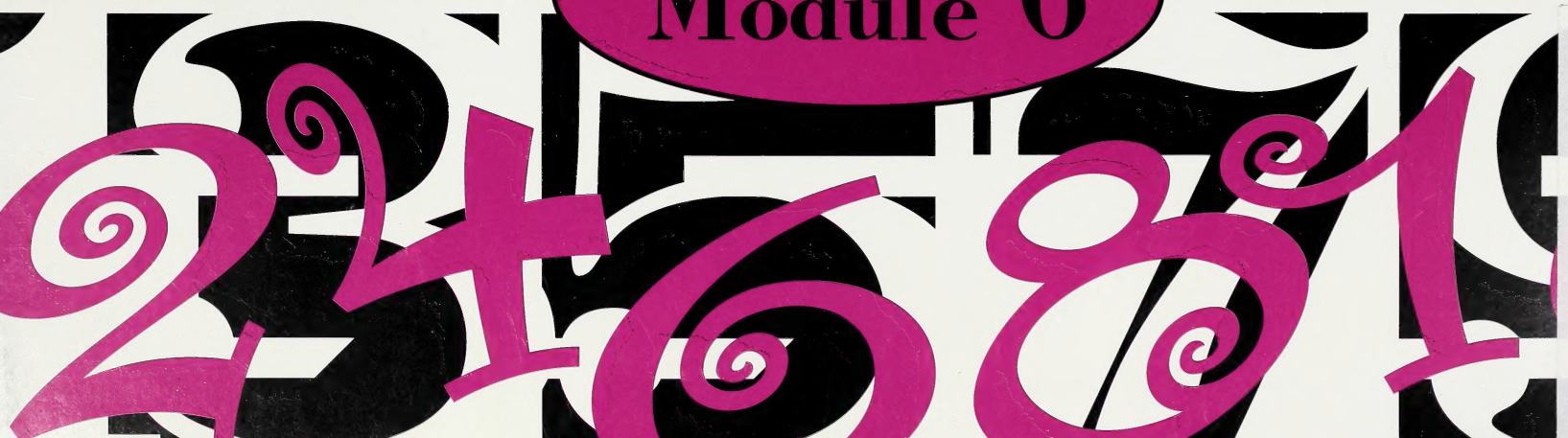
Module 6



Mathematics
Grade: 8
Basic

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Mathematics 8

Module 6

Three-Dimensional Geometry

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Mathematics 8
Student Module Booklet
Module 6
Three-Dimensional Geometry
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

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http://ednet.edc.gov.ab.ca/ltb

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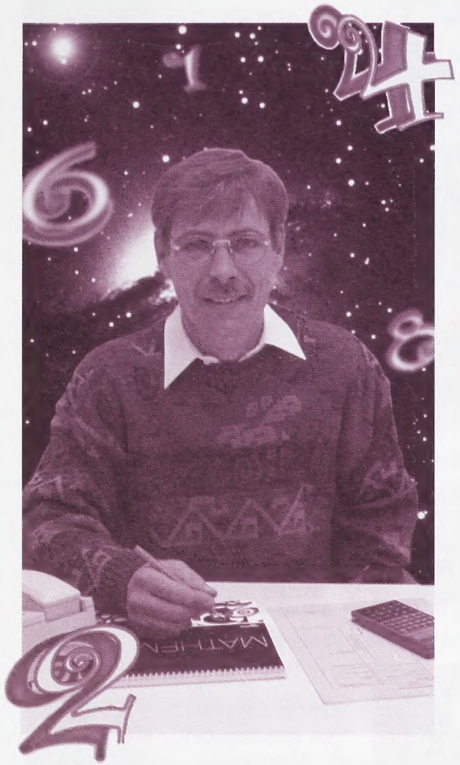
Welcome



Welcome to Module 6. We hope you'll enjoy your study of Three-Dimensional Geometry.

Mathematics 8 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

Mathematics 8

The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

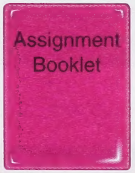
- Use computer software.

- Use a scientific calculator.

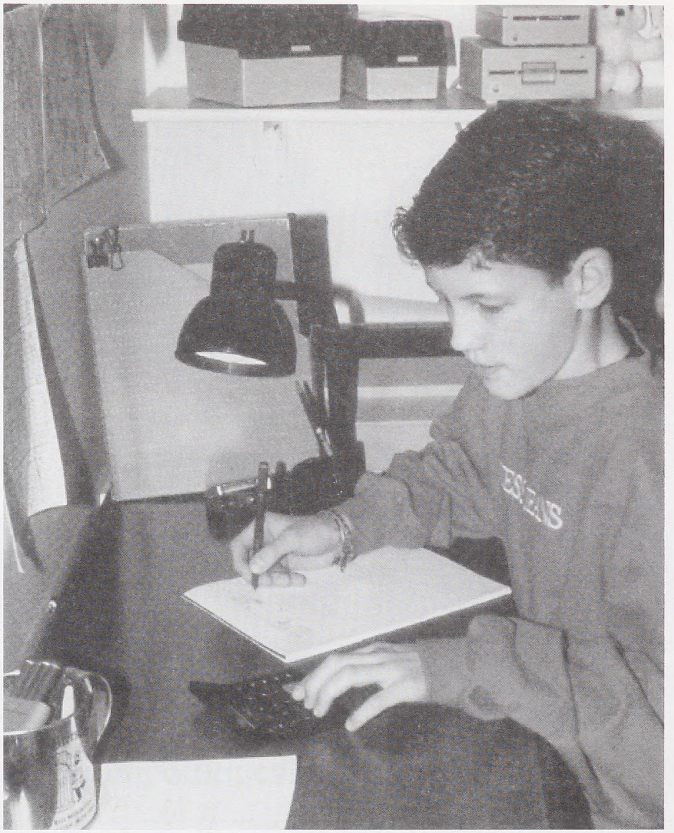
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Technology

Today society is turning to **technology** more than ever before, and it is to your advantage to be able to effectively use technology when required.

Technology is the application of tools, materials, and processes to the solution of problems. More specifically, technology refers to devices and systems that are used in processing, transferring, storing, and communicating information through electronic media.

In Mathematics 8, along with the course materials, you will use a calculator, computer, and videocassette player as tools for learning and doing mathematics.

Calculators are helpful tools for solving problems and exploring patterns and relationships between numbers. Using a calculator will also save you time and help you develop your estimating skills. Therefore, you will be given numerous opportunities in each module to use a calculator.

Computers are useful for organizing and displaying data, or drawing figures. For this reason you will have the chance in many activities to work with popular computer applications such as spreadsheets and draw programs. You will also want to check out the many Internet connections in each module.

Videocassette players allow you to view video programs on key concepts that are difficult to explain in print. That is why video programs are cited in this course.

It is expected that all of you will be able to view the video programs and use a calculator, and that most of you will do the computer activities. However, if you are unable to access a computer, you may do the calculations using a calculator, and draw figures and graphs by hand.

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge. Watch for these icons.

This icon is a cue that the problem will be related to the topic of the activity.

This icon is a cue that the problem will provide a change of topic.

The Four-Stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem. You may consider the following strategies:

- using objects
- using diagrams
- making a table
- working backwards
- using elimination
- using truth tables
- using an equation
- changing your point of view
- making an organized list
- using Venn diagrams
- simplifying a problem
- guessing, checking, and revising
- finding and applying a pattern
- acting out a problem

Note: The Appendix in this module explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Module 6 Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

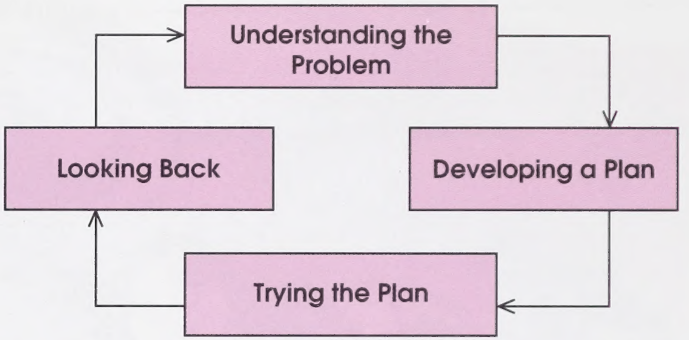
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

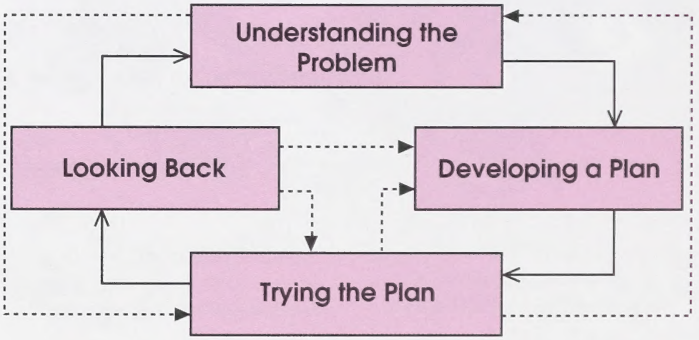
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



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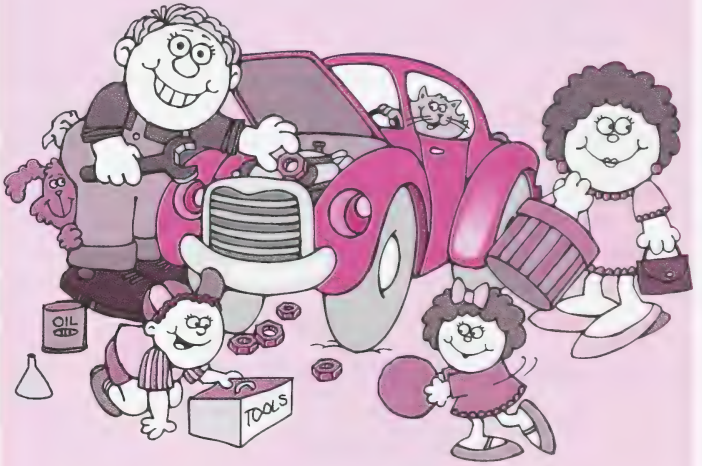
Module Overview

Drawings are flat, two-dimensional (2-D) copies of a three-dimensional (3-D) world. Real people and objects, unlike drawings, have depth as well as length and width.

Many of the objects in the everyday world are common geometric shapes. A ball is a sphere. Most boxes are cubes or prisms. A tin can is a cylinder. Other objects are composite shapes. A funnel is a combination of two shapes—a cone and a cylinder. A bolt is made up of two cylinders. A nut is a prism minus a cylinder.

In this module you will investigate three-dimensional geometry. You will build many 3-D shapes and explore their properties. You will generalize measurement patterns and procedures. You will solve problems involving volume and surface area.

Module 6
Three-Dimensional Geometry



Section 1
Properties of
Three-Dimensional Shapes

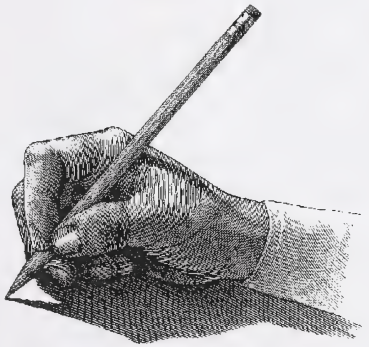
Section 2
Measurement of
Three-Dimensional Shapes

Evaluation

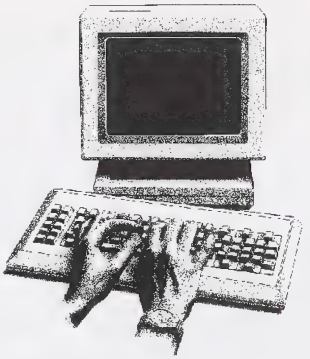
Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete three assignments. The mark distribution is as follows:

Section 1 Assignment	**40 marks**
Section 2 Assignment	**45 marks**
Final Module Assignment	**15 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.



If you are working on a computer managed learning (CML) terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: Properties of Three-Dimensional Shapes

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Science World in Vancouver, B.C., is a place where you may create a cyclone, crawl through a beaver lodge, dance on a giant keyboard, or blow square bubbles. You can also experience a sensational OMNIMAX® film.

The building which houses Science World was originally built for Expo 86. It is a geodesic dome made up of 766 triangles. Geodesic domes were invented by R. Buckminster Fuller in 1951. These three-dimensional shapes are very strong and require no load-bearing interior walls to support their roofs. They are also very energy-efficient because a maximum amount of space is enclosed with a minimum amount of materials.

In this section you will build many three-dimensional shapes and explore their properties.

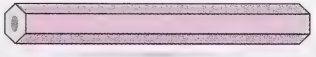
Activity 1: Polyhedra

What objects shown in this diagram have one or more flat surfaces?

The suitcase, the tennis racket, the picnic basket, and the goggles are some of the objects that have one or more flat surfaces.

A three-dimensional shape with **only** flat surfaces is called a **polyhedron**. **Note:** The plural of polyhedron is **polyhedra**.

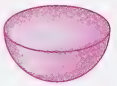
An unsharpened pencil is a polyhedron because **all** its surfaces are flat.



A can of paint is **not** a polyhedron because some of its surfaces are curved.

1. Is each of the following shapes a diagram of a polyhedron? Answer **yes** or **no**.

a.



b.

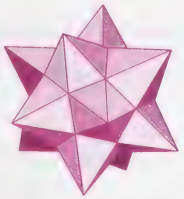
c.

d.

e.

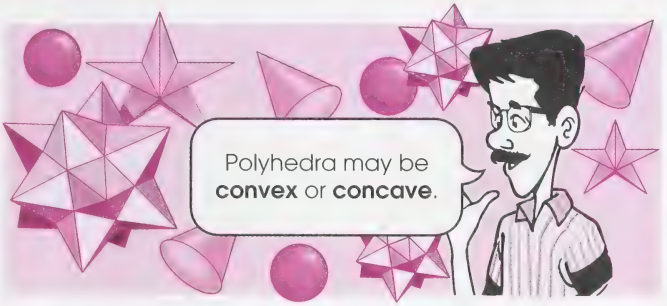
f.

g.



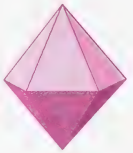
h.

Check your answers by turning to the Appendix.



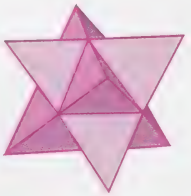
In a convex polyhedron, all the flat surfaces slope outwards.

This is an example of a convex polyhedron.



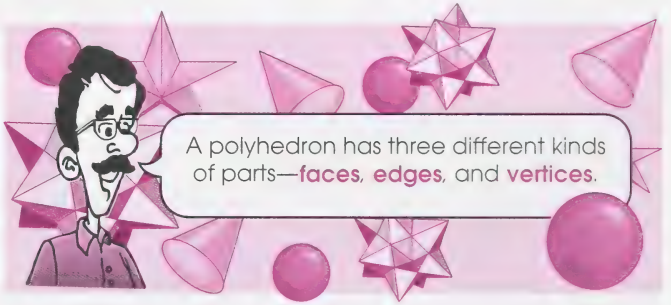
In a concave polyhedron, some of the flat surfaces slope inwards.

This is an example of a concave polyhedron.



2. State whether each of the polyhedra that you identified in question 1 is concave or convex. **Note:** Write N/A (for *not applicable*) if the shape is not a polyhedron.

Check your answers by turning to the Appendix.



A face is one of the flat surfaces. An edge is a line segment formed by the intersection of two faces. A vertex is a point where three or more edges meet. **Note:** The plural of vertex is **vertices**.

It is easier to examine the faces, edges, and vertices of a polyhedron if you examine a model rather than a diagram.

In this activity you will use **nets** to make the **shells** of various polyhedra.

A net is a two-dimensional pattern that can be folded into a three-dimensional shape. A shell is a model in which the interior is empty.

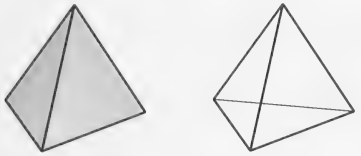
You will also make the **skeletons** of various polyhedra.

A skeleton is a three-dimensional model showing only the edges and vertices.

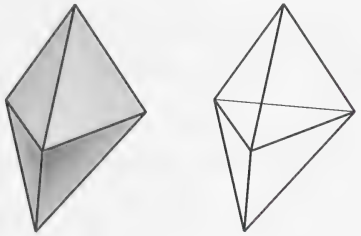
To make a skeleton, you can use toothpicks joined together with mini-marshmallows or bits of Plasticine™ or modelling clay. Alternatively, you can use pieces of drinking straws joined together with pipe cleaners.

Following are diagrams showing the shells and skeletons of three polyhedra with triangular faces. Notice that a shell emphasizes the flat surfaces or faces and a skeleton emphasizes the edges and vertices.

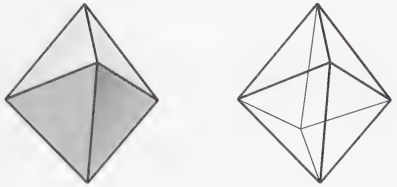
- This polyhedron has four triangular faces.



- This polyhedron has 6 triangular faces.



- This polyhedron has 8 triangular faces.



In the Appendix, find nets 1 to 8 (notice the nets have identical equilateral faces). Photocopy these nets (if you wish, you may enlarge the nets). Glue the photocopies to heavier paper or card stock. Then cut out the nets along the solid lines.

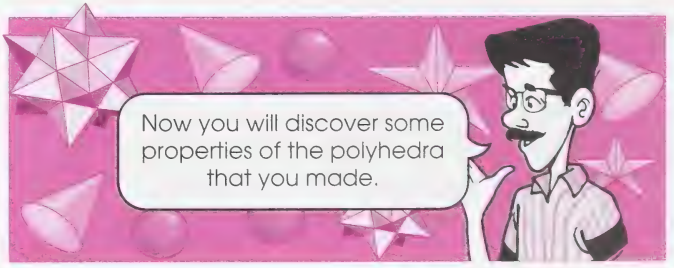
Assemble each of the eight nets to make shells using the following steps.

Step 1: Trace the dotted lines in the net using a ruler and the point of a pair of compasses. Be sure to press firmly. This process is known as **scoring**.

Step 2: Fold the net along the dotted lines. Be sure the dotted lines are on the outside of the creases.

Step 3: Tape the faces together.

Next, make a skeleton of each polyhedron that you have constructed.



3. Is each of the polyhedra that you have modelled convex?

4. The polyhedron that you made from Net 1 has 4 faces.

   a. How many vertices does it have? How many edges does it have?

   b. How many faces meet each vertex? How many edges meet at each vertex?

5. The polyhedron that you made from Net 2 has 6 faces.

   a. How many vertices does it have? How many edges does it have?

   b. How many faces meet each vertex? How many edges meet at each vertex?

6. The polyhedron that you made from Net 3 has 8 faces.

   a. How many vertices does it have? How many edges does it have?

   b. How many faces meet each vertex? How many edges meet at each vertex?

7. The polyhedron that you made from Net 4 has 10 faces.

   a. How many vertices does it have? How many edges does it have?

   b. How many faces meet each vertex? How many edges meet at each vertex?

8. a. Use your answers to questions 4 to 7 to complete a table like the following.

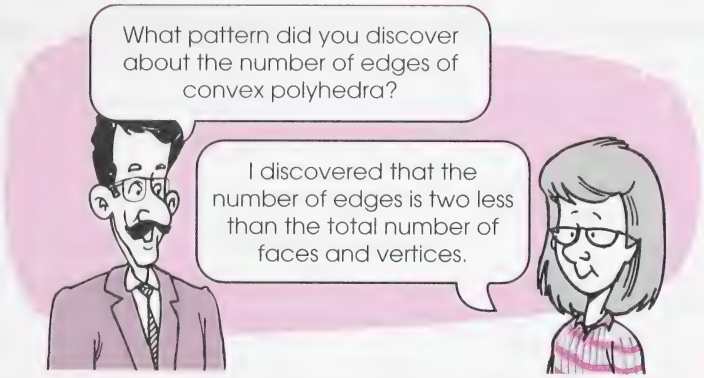
Convex Polyhedra			
Number of Faces	Number of Vertices	Total Number of Faces and Vertices	Number of Edges
4			
6			
8			
10			

   b. How is the number of edges of each convex polyhedron related to the total number of faces and vertices?

   c. Write a formula to describe the relationship between the number of edges of each convex polyhedron and the total number of faces and vertices.

   Let f be the number of faces.
   Let v be the number of vertices.
   Let e be the number of edges.

Check your answers by turning to the Appendix.
Save the models; you will use them again.


What pattern did you discover about the number of edges of convex polyhedra?
I discovered that the number of edges is two less than the total number of faces and vertices.

Use the formula $e = f + v - 2$ to answer the following questions.

9. **a.** The polyhedron made from Net 5 has 12 faces and 8 vertices. How many edges does it have?

b. The polyhedron made from Net 6 has 14 faces and 9 vertices. How many edges does it have?

c. The polyhedron made from Net 7 has 16 faces and 24 edges. How many vertices does it have?

d. The polyhedron made from Net 8 has 12 vertices and 30 edges. How many faces does it have?

Check your answers by turning to the Appendix. You may also use the models that you have made to check your answers.

A regular polyhedron is a polyhedron in which all the faces are identical regular polygons and in which the same number of faces and the same number of edges meet at each vertex.

10. Examine the polyhedra that you made from nets 1 to 8. Notice that each polyhedron has only regular equilateral faces.

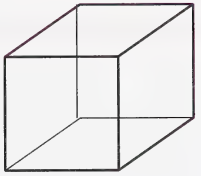
Which of these polyhedra are regular polyhedra?

Check your answer by turning to the Appendix.

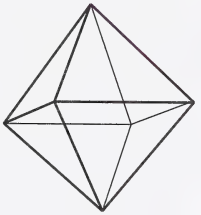
There are only five regular polyhedra. Each of these five shapes is named according to its number of faces.

- A regular **tetrahedron** has 4 faces (each face is an equilateral triangle).

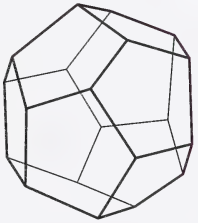
- A regular **hexahedron** has 6 faces (each face is a square). **Note:** A regular hexahedron is also called a **cube**.



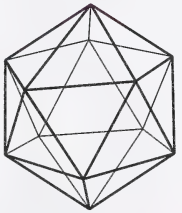
- A regular **octahedron** has 8 faces (each face is an equilateral triangle).

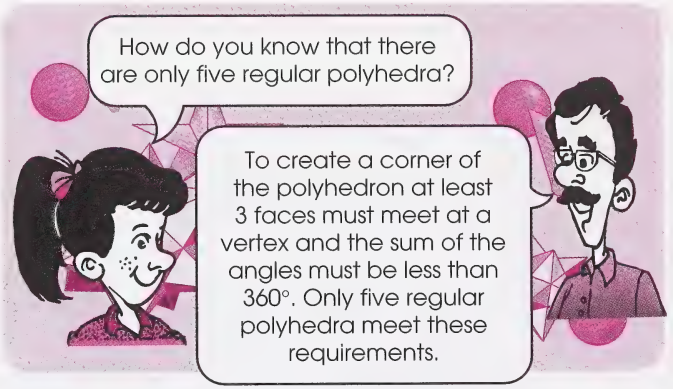


- A regular **dodecahedron** has 12 faces (each face is a regular pentagon).



- A regular **icosahedron** has 20 faces (each face is an equilateral triangle).




How do you know that there are only five regular polyhedra?
To create a corner of the polyhedron at least 3 faces must meet at a vertex and the sum of the angles must be less than 360°. Only five regular polyhedra meet these requirements.

In the Appendix, find the pages of regular polygons. Photocopy each page, glue the photocopies to heavy paper or card stock, and cut out the regular polygons.

Use the regular polygons to answer questions 11 to 14. **Note:** Because each vertex of a regular polyhedron has the same number of faces and the same number of edges, you will attempt to build only one corner of a polyhedron. Tape the given number of polygons together and fold the edges.

11. a. Can you create a corner (vertex) of a regular polyhedron with 3 equilateral triangles? Why or why not?

b. Can you create a corner of a regular polyhedron with 4 equilateral triangles? Why or why not?

c. Can you create a corner of a regular polyhedron with 5 equilateral triangles? Why or why not?

d. Can you create a corner of a regular polyhedron with 6 equilateral triangles? Why or why not?

e. Can you create a corner of a regular polyhedron with 7 equilateral triangles? Why or why not?

f. Based on your experiment, how many regular polyhedra can be formed with equilateral triangles?

12. a. Can you create a corner of a regular polyhedron with 3 squares? Why or why not?

b. Can you create a corner of a regular polyhedron with 4 squares? Why or why not?

c. Can you create a corner of a regular polyhedron with 5 squares? Why or why not?

d. Based on your experiment, how many regular polyhedra can be formed with squares?

13. a. Can you create a corner of a regular polyhedron with 3 regular pentagons? Why or why not?

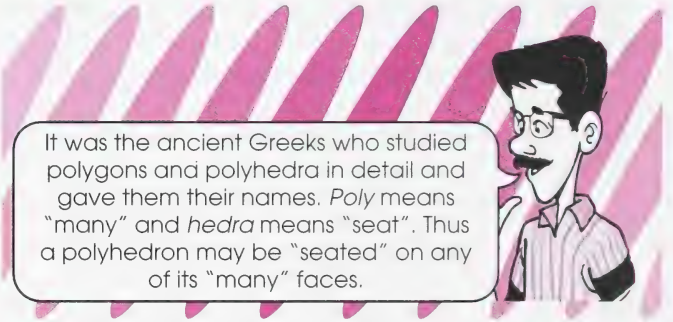
b. Can you create a corner of a regular polyhedron with 4 regular pentagons? Why or why not?

c. Based on your experiment, how many regular polyhedra can be formed with regular pentagons?

14. a. Can you create a corner of a regular polyhedron with 3 regular hexagons? Why or why not?

b. Based on your experiment, how many regular polyhedra can be formed with regular hexagons?

Check your answers by turning to the Appendix.


It was the ancient Greeks who studied polygons and polyhedra in detail and gave them their names. *Poly* means "many" and *hedra* means "seat". Thus a polyhedron may be "seated" on any of its "many" faces.

When a polyhedron is made up of identical faces, any face can be the **base**.

The base of a polyhedron is the face from which the **height** is measured. The height of a polyhedron is the vertical distance from a vertex to the base (or extension of the base).

15. a. Examine the regular polyhedra that you made from nets 1, 3, and 8. Does the height of each polyhedron appear to be the same no matter on which face the polyhdron is seated?

b. Examine the irregular polyhedra that you made from nets 2, 4, 5, 6, and 7. Does the height of each polyhedron appear to be the same no matter on which face the polyhdron is seated?

Check your answers by turning to the Appendix.

You will now investigate a **semi-regular polyhedron**.

A semi-regular polyhedron is a polyhedron in which the faces consist of at least two different types of regular polygons combined in the same way around each vertex.

A soccer ball is an example of a semi-regular polyhedron.

16. **a.** Which regular polygons are on the surface of a soccer ball?

b. How many regular polygons are at each vertex of a soccer ball? How many edges are at each vertex?

c. Does a soccer ball have the same height no matter on what face it is seated?

Check your answers by turning to the Appendix.

You will now examine irregular polyhedra which have faces of different shapes and sizes.

In the Appendix, find nets 9 to 12. Photocopy the nets and glue the copies to heavy paper or card stock. Cut out each of the nets along the solid lines.

Use the nets to assemble the polyhedra.

17. Examine the polyhedron that was made from Net 9.

a. What kinds of faces does it have?

b. Does the height of the polyhedron vary depending on which face it is seated?

18. Examine the polyhedron that was made from Net 10.

a. What kinds of faces does it have?

b. Does the height of the polyhedron vary depending on which face it is seated?

19. Examine the polyhedron that was made from Net 11.

a. What kinds of faces does it have?

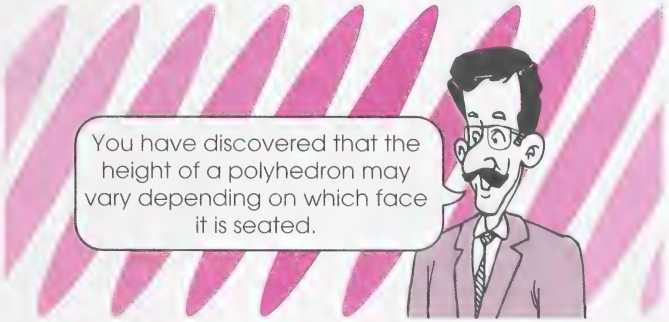
b. Does the height of the polyhedron vary depending on which face it is seated?

20. Examine the polyhedron that was made from Net 12.

a. What kinds of faces does it have?

b. Does the height of the polyhedron vary depending on which face it is seated?

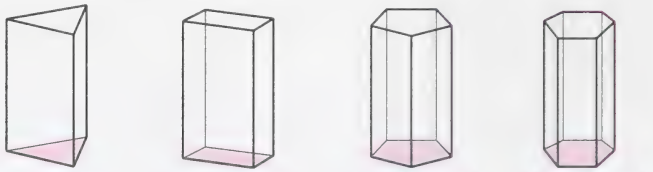
Check your answers by turning to the Appendix. Save the polyhedra you have constructed; you will use them again.



For this reason, one face or a pair of faces may be designated as a base and the other faces may be designated as "side" faces.

The base of a **right prism** may be any kind of polygon, but the "side" faces (called the lateral faces) are always rectangles. Right prisms are named according to the shape of their bases.

Following are examples of right prisms.



Note: When each face of a rectangular prism is a square, the prism is called a **cube**.

The base of a **right pyramid** may be any kind of polygon, but the "side" faces (called the lateral faces) are always triangles. Right pyramids are named according to the shape of their bases.

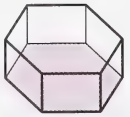
Following are examples of right pyramids.

Note: When the base of a rectangular pyramid is a square, the pyramid is called a **square pyramid**.

21. State whether each of the following right prisms is seated on a base or a "side" face.

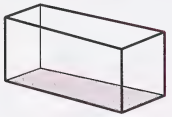
a.

b.



c.

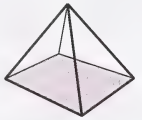
d.



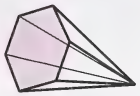
22. State whether each of the following pyramids is seated on a base or a "side" face.

a.

b.



c.



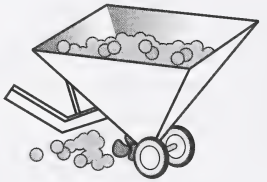
d.

23. **a.** Name each of the right prisms in question 21.
b. Name each of the pyramids in question 22.
c. Name each of the polyhedra you made from nets 9 to 12.

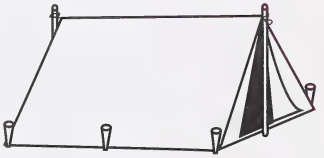
Check your answers by turning to the Appendix.

24. State whether each of the following objects resembles a right prism or a pyramid.

a. this fertilizer spreader



b. this tent



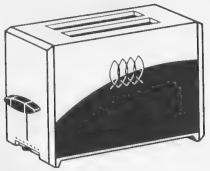
c. the roof of this gazebo

d. the roof of the tower of this building

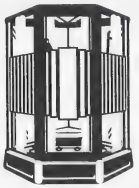
e. this box of disposable towels

f. this piece of cheese

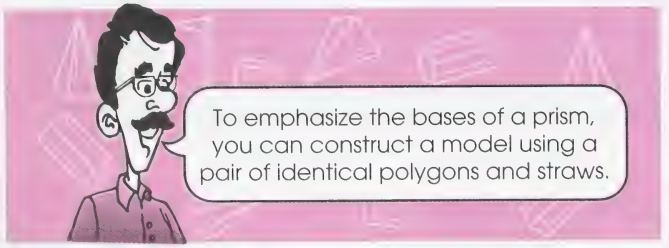
g. this toaster



h. this shower



Check your answers by turning to the Appendix.

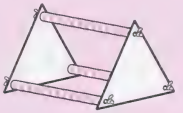


To make a model of a triangular prism, follow these steps.

Step 1: Cut out two identical triangles from card stock. Punch holes in the corners of each triangle.

Step 2: Cut 3 pieces of plastic straws the same length. Thread a piece of yarn or string through each straw and then through corresponding holes in the corners of each triangle.

Step 3: Knot each piece of string to hold the model together.



Gather straws, string, and regular polygons (you may photocopy the regular polygons in the Appendix, glue the photocopies to heavy paper or card stock, and cut out the regular polygons). Then make models of a triangular prism, a pentagonal prism, and a hexagonal prism.

Use the models you have constructed to answer question 25.

25. **a.** Is each pair of opposite faces in a prism parallel?

b. Is each "side" face in a prism perpendicular to the base?

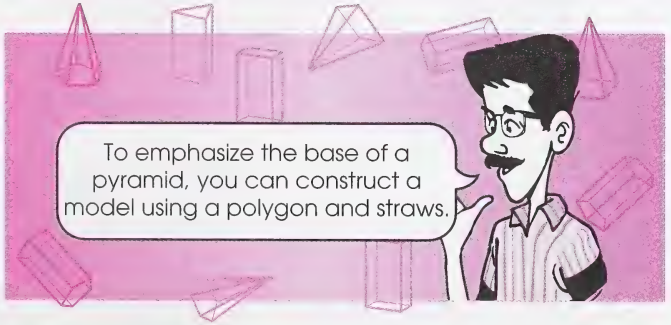
c. Do the same number of faces meet at each vertex?

d. Do the same number of edges meet at each vertex?

e. Is the number of edges two less than the sum of the number of faces and vertices?

f. If you know the number of sides on the base of a prism, how can you calculate the number of edges? the number of vertices?

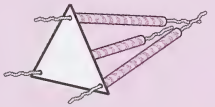
Check your answers by turning to the Appendix.



To construct a model of a triangular pyramid, follow these steps.

Step 1: Cut out a triangle from card stock. Punch holes in the corners.

Step 2: Cut 3 pieces of plastic straws the same length. Thread a piece of yarn or string through each straw and then through a hole in the triangle. Knot each string.



Step 3: Bring the straws together at the opposite end. Knot the ends of each pair of strings.

Gather regular straws, string, and regular polygons (you may photocopy the regular polygons in the Appendix, glue the photocopies to heavy paper or card stock, and cut out the regular polygons). Then make models of a triangular pyramid, a pentagonal pyramid, and a hexagonal pyramid.

Use the models you have constructed to answer question 26.

26. **a.** Is each "side" face in a pyramid perpendicular to the base?

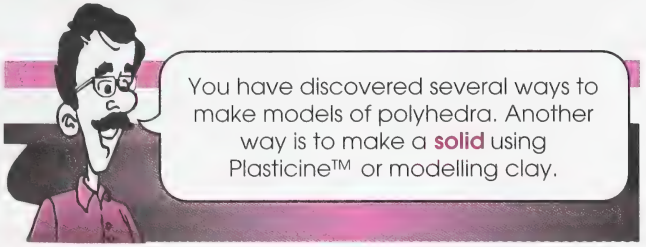
b. Do the same number of faces meet at each vertex?

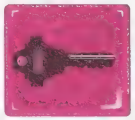
c. Do the same number of edges meet at each vertex?

d. Is the number of edges two less than the sum of the number of faces and vertices?

e. If you know the number of sides on the base of a pyramid, how can you calculate the number of edges? the number of vertices?

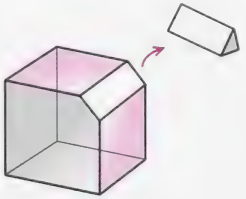
Check your answers by turning to the Appendix.


You have discovered several ways to make models of polyhedra. Another way is to make a solid using Plasticine™ or modelling clay.



A solid is a three-dimensional model in which the interior is filled.

You can create a **cross section** by cutting a solid with a knife. For example, cutting a cube at the given angle creates a rectangular cross section.



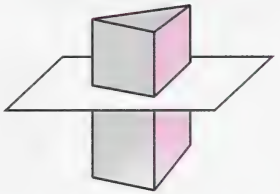
cube cross section

A cross section is a two-dimensional figure that is the intersection of a three-dimensional figure and a **plane**. A plane is a flat surface.

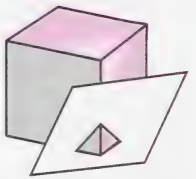
27. Sketch the cross section formed by the intersection of each plane and polyhedron. **Note:** Each plane acts like a knife.

Hint: To help you visualize each cross section, you may wish to model each polyhedron with clay and make a cut as indicated.

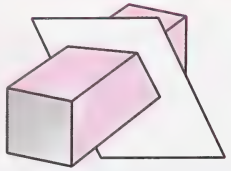
a. Triangular Prism



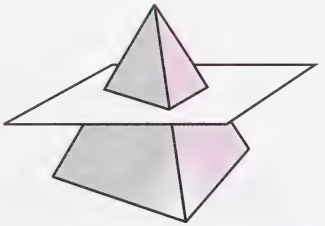
b. Cube



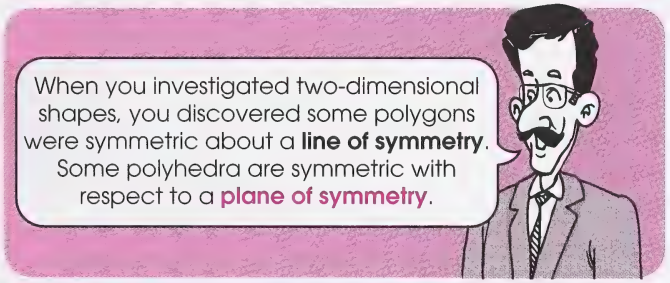
c. Rectangular Prism



d. Square Pyramid



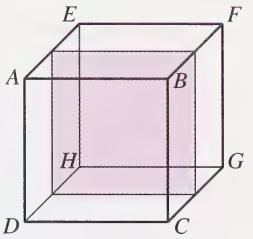
Check your answers by turning to the Appendix.


When you investigated two-dimensional shapes, you discovered some polygons were symmetric about a **line of symmetry**. Some polyhedra are symmetric with respect to a **plane of symmetry**.

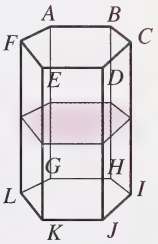
A plane of symmetry is a plane (flat surface) which divides a symmetical three-dimensional figure into two identical halves.

A three-dimensional shape is symmetric with respect to a plane of symmetry if, and only if, every point of the figure on one side of the plane is balanced by a symmetrical point on the opposite side of the plane.

The coloured plane in the diagram to the right is a plane of symmetry of the cube. It divides the cube into two identical pieces. Point A balances point E, point B balances point F, and so on.

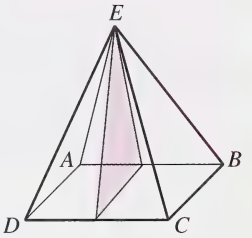


28. The coloured plane in the given diagram is a plane of symmetry of the hexagonal prism.



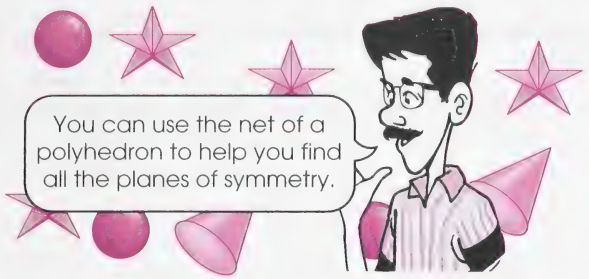
a. What point is balanced by A?
b. What point is balanced by B?
c. What point is balanced by C?
d. What point is balanced by D?
e. What point is balanced by E?
f. What point is balanced by F?

29. The coloured plane in the diagram to the right is a plane of symmetry of the square pyramid.



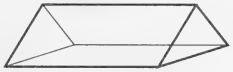
a. What point is balanced by A?
b. What point is balanced by D?

Check your answers by turning to the Appendix.



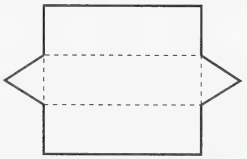
Example

How many planes of symmetry are in this triangular prism? **Hint:** The base is an equilateral triangle.



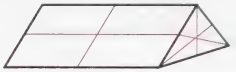
Solution

Step 1: Obtain the net of a triangular prism.



Step 2: On each region of the net of a triangular prism, draw all the lines of symmetry. **Hint:** An equilateral triangle has 3 lines of symmetry. A rectangle has 2 lines of symmetry.

Step 3: Assemble the net into a triangular prism with the lines of symmetry on the outside.



Step 4: Use the lines of symmetry to help you identify each plane of symmetry.

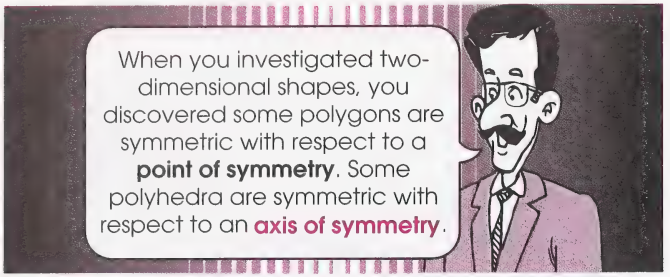
A triangular prism has four planes of symmetry.

30. **a.** How many planes of symmetry does a square pyramid have? **Hint:** The net of this shape is in the Appendix. See Net 10.

b. How many planes of symmetry does a rectangular prism have? **Hint:** The net of this shape is in the Appendix. See Net 9.

c. How many planes of symmetry does a cube have?

Check your answers by turning to the Appendix.

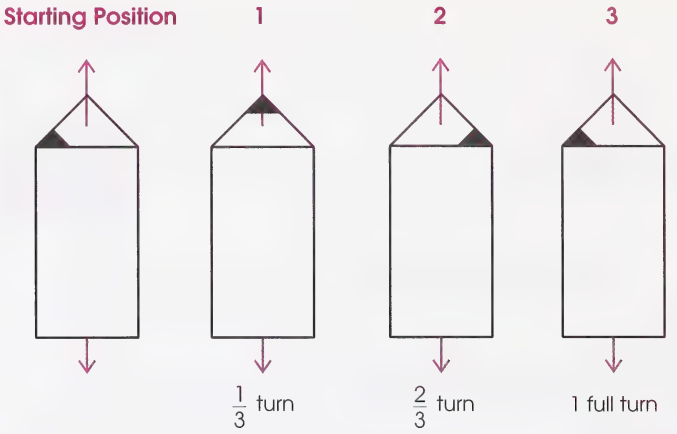

When you investigated two-dimensional shapes, you discovered some polygons are symmetric with respect to a **point of symmetry**. Some polyhedra are symmetric with respect to an **axis of symmetry**.

An axis of symmetry is a line about which a three-dimensional figure can be turned so that its turn image matches its original position.

A polyhedron is symmetric with respect to an axis of symmetry, if and only if, the turn image will coincide with the original position more than once in a full turn.

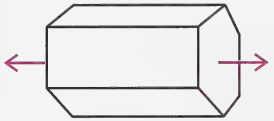
For example, the line in this diagram represents an axis of symmetry for the triangular prism. **Note:** The base is an equilateral triangle.

If the triangular prism is turned about this axis of symmetry it will be in a matching position 3 times in a full turn.

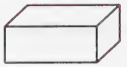


31. a. The line in this diagram represents an axis of symmetry for the square pyramid. How many times will the square pyramid be in a matching position in a full turn? **Hint:** Use the model of a square pyramid to help you.

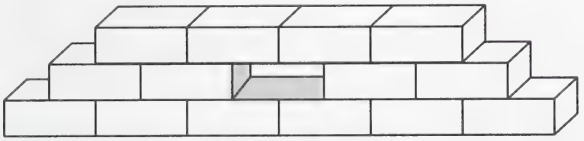
b. The line in this diagram represents an axis of symmetry for the hexagonal prism. How many times will the hexagonal prism be in a matching position in a full turn? **Hint:** Use the model of a hexagonal prism to help you.



32. This brick is a rectangular prism. **Note:** The ends are rectangular faces.



In how many ways can the brick be fitted into the space shown in this diagram?

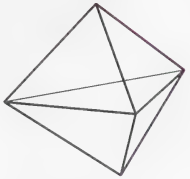


Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

33. Each edge of the regular tetrahedron that you made from Net 1 is 5 cm long. What are the dimensions of the smallest square hole that will allow this figure to pass through it?



Check your answer by turning to the Appendix.

Did You Know?

Many beautiful kinds of symmetry occur in nature. For example, diamonds, sulphur, and quartz crystals are symmetrical.

Diamond

Sulphur

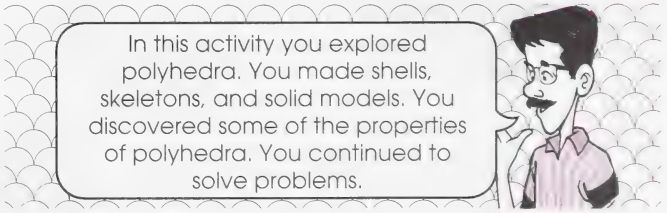
Quartz

A Canadian mathematician has helped crystallographers understand how the atoms of crystals are arranged to create these shapes.

In the Appendix, find and read the article "Coxeter Groups."

Use the Internet to find out more about Dr. Coxeter. This is the URL (uniform resource locator) of a site which shows a photo of Dr. Coxeter and models of polyhedra.

http://www.li.net/~george/coxeter.html


In this activity you explored polyhedra. You made shells, skeletons, and solid models. You discovered some of the properties of polyhedra. You continued to solve problems.

Activity 2: Spheres, Cones, and Cylinders

What objects shown in this diagram have one or more curved surfaces?

The beach ball has a curved surface; it is a **sphere**. The party hat has a curved surface; it is a **cone**. The pool has a curved surface; it is a **cylinder**.

In this activity you will explore spheres, cones, and cylinders.

1. State whether each of the following resembles a sphere, a cone, or a cylinder.

a. a drum

b. the roof on the turret of this Victorian house

c. a soda can

d. a grape

e. a spruce tree

f. a hay bale

2. Peel the label from an empty can of food. What is the shape of the label? What does this indicate about the curved surface of a cylinder?

3. Cut a conical party hat or conical drinking cup up one side and then flatten the hat or cup into a 2-D shape. What shape is the flattened hat or cup? What does this indicate about the curved surface of a cone?

Check your answers by turning to the Appendix.

Were you surprised to discover that the curved surface of a cylinder is a rectangle and the curved surface of a cone is a sector of a circle?

Make a number of cones from circles by following these steps.
Note: Use different sizes of central angles.

Step 1: Draw a circle using a pair of compasses.

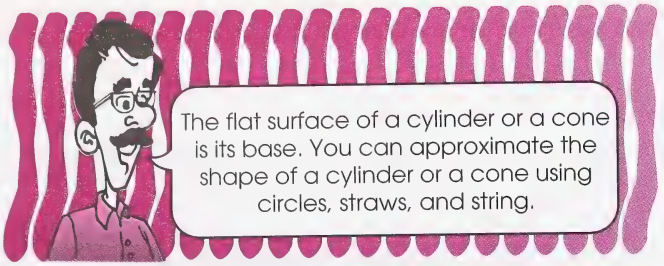
Step 2: Draw a central angle.

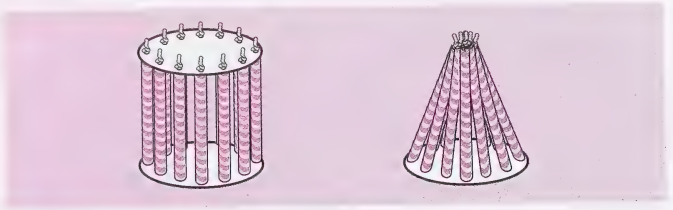
Step 3: Cut the circle along the central angle to create 2 pieces.

Step 4: Tape the straight edges of each piece together to create a pair of cones.

4. How does the size of the central angle affect the height and diameter of the cone?

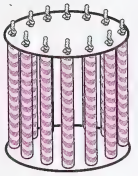
Check your answer by turning to the Appendix.


The flat surface of a cylinder or a cone is its base. You can approximate the shape of a cylinder or a cone using circles, straws, and string.



5. **a.** In what way is a cylinder similar to a prism?
   b. In what way is a cone similar to a pyramid?

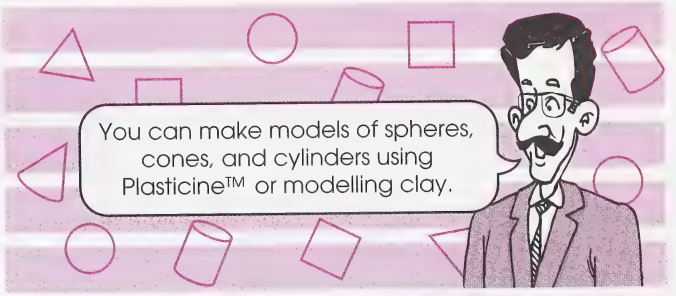
6. Suppose you wrapped a rectangle around this model of a cylinder to cover the "sides".



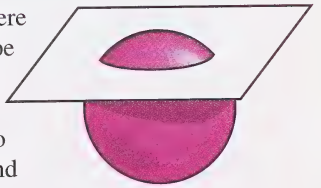
   a. How is the length of the rectangle related to the circumference of the circular base of the cylinder?

   b. How is the width of the rectangle related to the height of the cylinder?

Check your answers by turning to the Appendix.



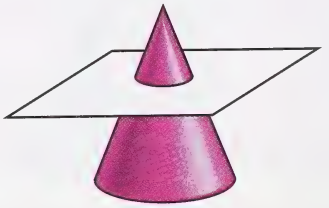
7. **a.** Imagine that you sliced a sphere at this angle. What is the shape of the cross section? **Hint:** To help you visualize the cross section you may wish to model the sphere with clay and make a cut as indicated.



   b. Is there a way to cut a sphere so that the cross section is a different shape?

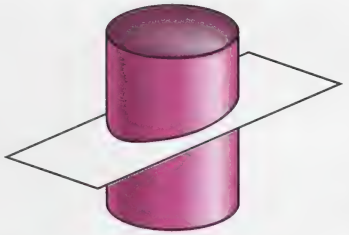
8. Sketch the cross section formed by the intersection of each plane and three-dimensional shape. **Hint:** To help you visualize each cross section, you may wish to model each shape with clay and make a cut as indicated.

   a.

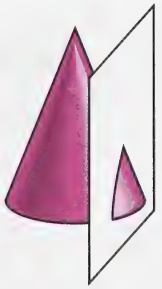


b.

c.



d.

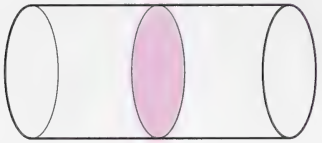


Check your answers by turning to the Appendix.

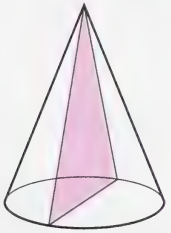
9. Is each of these coloured planes a plane of symmetry for the object shown?

a.

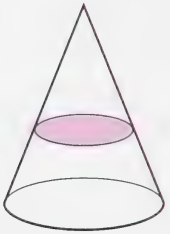
b.



c.

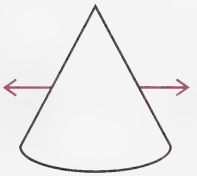


d.

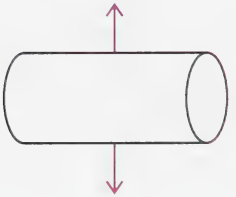


10. Is each of these lines an axis of symmetry for the object shown?

a.



b.



c.

d.

Check your answers by turning to the Appendix.

Earth is a sphere. As Earth rotates on an axis of symmetry, the rotation causes night and day.

11. Through what points on Earth does this axis of symmetry pass?
Hint: The photograph of the globe may help you answer this question.

Check your answer by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer each of the following questions.

12. Ruth surveyed 75 students about what sports they like. Of the students surveyed, most students liked basketball, football, or baseball.

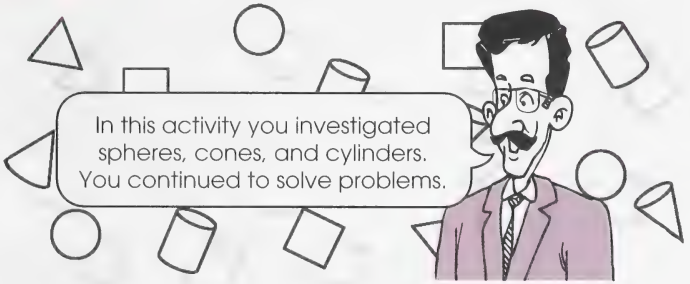
- 48 like basketball.
- 45 like football.
- 58 like baseball.
- 28 like basketball and football.
- 37 like football and baseball.
- 40 like basketball and baseball.
- 25 students like basketball, football, and baseball.

a. How many students like a sport other than basketball, football, and baseball?

b. What is the probability that a student chosen at random would like basketball, football, or baseball?

13. How can five matches be arranged so that each match touches every other match?

Check your answers by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

To conserve space, moving companies store packing boxes flat. The boxes are then put together as they are needed. The flat boxes can be thought of as nets which are assembled into rectangular prisms.

In this section you used **nets** to create models of various 3-D shapes. You also explored the properties of 3-D shapes by building **skeletons** and other models.

To review what you have discovered about the surface of three-dimensional figures, answer the following questions.

1. State whether each of the following statements is true or false. Write **T** or **F**.

a. All the faces of a triangular prism are rectangles.

b. A rectangular prism has six faces.

c. A triangular prism has six faces.

d. A sphere has no base.

e. The bases of a triangular prism are triangles.

f. A pyramid has two bases.

g. A cylinder has parallel bases.

h. All prisms have six faces.

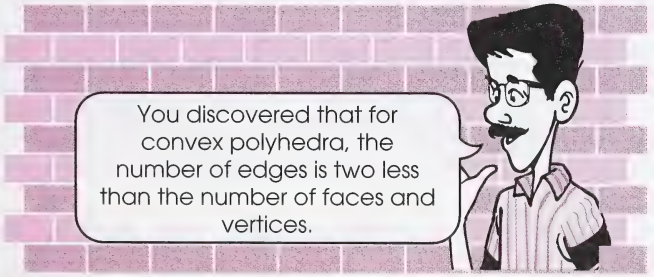
i. All prisms have two bases.

j. All pyramids have triangular "side" faces.

k. The bases of all pyramids are polygons.

l. The curved surface of a cylinder when flattened is a rectangle.

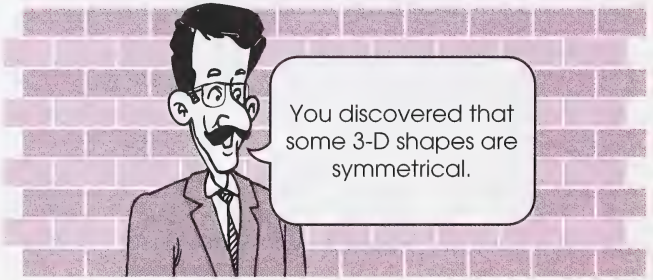
Check your answers by turning to the Appendix.



2. Complete a table like the following.

Shape	Number of Faces	Number of Vertices	Number of Edges
Triangular Prism			
Triangular Pyramid			
Cube			
Square Pyramid			
Hexagonal Prism			
Hexagonal Pyramid			

Check your answers by turning to the Appendix.



3. **a.** What is a plane of symmetry?
   b. What is an axis of symmetry?

Check your answers by turning to the Appendix.

Enrichment

In this Enrichment you will explore **star-shaped polyhedra**, also called **stellated polyhedra** (*stella* is the Latin word for star).

This stellated octahedron was made from a regular octahedron. **Note:** The pyramid additions are extensions of the faces of the regular polyhedron.

Use the Internet to find diagrams of various stellated polyhedra. This is the uniform resource locator (URL) of a site you may find helpful.

http://www.teleport.com/~tpgettys/stellate.shtml

To discover more about star-shaped polyhedra, you will build a model of a small stellated dodecahedron.

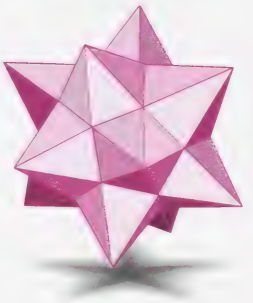
In the Appendix, find Net 13. Photocopy the page, glue the copy to heavy paper or card stock, and cut out the net.

Assemble a dodecahedron from Net 13. Use the model to answer the following question.

1. **a.** Is this dodecahedron convex or concave?
   b. Is this dodecahedron symmetric?

In the Appendix, find the 12 copies of Net 14. Photocopy the pages, glue the photocopies to heavy paper or card stock, and cut out the nets.

Assemble 12 pyramids from the 12 copies of Net 14. Then tape each pyramid to a face of the dodecahedron to make a stellated docecahedron. **Note:** The faces of each pyramid are extensions of the faces of the dodecahedron. Use the model to answer the following question.



2. **a.** Is this stellated dodecahedron convex or concave?

   b. Is this stellated dodecahedron symmetric?

Check your answers by turning to the Appendix.

Conclusion

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In this section you investigated three-dimensional geometry. You built many three-dimensional shapes using card stock, drinking straws, pipe cleaners, string, toothpicks, mini-marshallows, and Plasticine™ or modelling clay. You explored many properties of regular polyhedra, prisms, pyramids, spheres, cones, and cylinders.

You discovered that some of the objects around you have only flat surfaces. Some objects have only curved surfaces and other objects have both flat and curved surfaces.

The Spaceship Earth pavilion at Epcot in Disneyworld is a geodesic dome. How is the Epcot geodesic dome different from the Science World geodesic dome? How are these geodesic domes similar?

Assignment

Assignment Booklet

You are now ready to complete the assignment for Section 1.

Section 2: Measurement of Three-Dimensional Shapes

PHOTO SEARCH LTD

When you bake a cake you want to be sure that the cake pan is large enough so that the cake batter does not overflow as it rises.

Bakers use cake pans of many different shapes. The cake pan in the photograph is cylindrical. What do you think the volume of this cake pan is? What do you think the surface area of the cake will be?

In this section you will generalize measurement patterns and procedures. You will solve problems involving volume and surface area.

Activity 1: Volume

In the moving business, the amount of space used is very important. Movers are very skilled at arranging the contents of the moving van so that the boxes take up the least amount of space possible.

The amount of space an object occupies is called its **volume**.

In previous grades you have calculated the volume of rectangular prisms. In this activity you will calculate the volume of many kinds of prisms. You will also calculate the volume of cylinders.

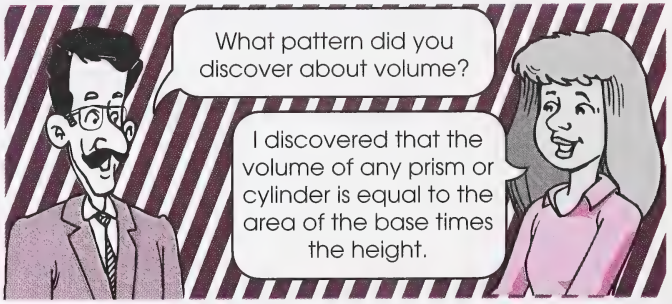
View the video program *Volume* from the series *Mathways*. It provides a good introduction to calculating the volume of prisms and cylinders.

Use what you learned in the video to answer question 1.

1. a. Explain why the volume of a rectangular prism can be expressed as the formula $V = \ell wh$, where V is the volume, ℓ is the length, w is the width, and h is the height of the rectangular prism.

b. Explain why the volume of a cylinder can be expressed by the formula $V = \pi r^2 h$, where V is the volume, r is the radius, and h is the height of the cylinder.

Check your answers by turning to the Appendix.


What pattern did you discover about volume?
I discovered that the volume of any prism or cylinder is equal to the area of the base times the height.

This relationship can be expressed by the following formula, where V is the volume of any prism or cylinder, B is the area of the base of the prism or cylinder, and H is the height of the prism or cylinder.

$$V = BH$$

Note: Upper-case letters are used for the area of the base and the height of the prism or cylinder to distinguish them from the variables used in area formulas.

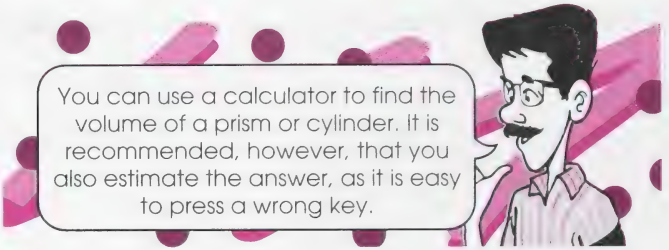
2. a. What does b represent in the formulas $A = bh$, $A = \frac{bh}{2}$, and $A = \frac{(b_1 + b_2)h}{2}$?

b. What does h represent in the formulas $A = bh$, $A = \frac{bh}{2}$, and $A = \frac{(b_1 + b_2)h}{2}$?

c. What does B represent in the formula $V = BH$?

d. What does H represent in the formula $V = BH$?

Check your answers by turning to the Appendix.


You can use a calculator to find the volume of a prism or cylinder. It is recommended, however, that you also estimate the answer, as it is easy to press a wrong key.

Some people prefer to calculate the answer first and then use estimation to determine if the answer is reasonable. Others prefer to estimate the answer first.

Example 1

This chocolate box is in the shape of a triangular prism.

[1]

The height of the box is 16.7 cm. The base of the box is a triangle with sides of 3 cm and a height of 2.5 cm. What is the volume of the box? **Note:** Round the answer to the nearest tenth.

[1] © Kraft Foods Canada, Inc. All rights reserved.

Solution

Step 1: Calculate the area of the base (B). **Note:** The base is a triangle.

$$B = \frac{bh}{2}$$
$$= \frac{3(2.5)}{2} \quad \leftarrow \text{Use a calculator.}$$
$$= 3.75$$

The area of the base (B) is 3.75 cm^2.

Step 2: Calculate the volume (V) of the prism.

$$V = BH$$
$$= 3.75(16.7) \quad \leftarrow \text{Use a calculator.}$$
$$= 62.625 \quad \leftarrow \text{Round to nearest tenth.}$$
$$\doteq 62.6$$

The volume (V) of the box is about 62.6 cm^3.

Note: Use estimation to determine if the answer is reasonable.

Rounding

$$B = \frac{bh}{2} \qquad V = BH$$
$$\doteq \frac{3(3)}{2} \qquad \doteq 4.5(20)$$
$$\doteq 4.5 \qquad \doteq 90$$

The answer is reasonable.

Front-end Digits

$$B = \frac{bh}{2} \qquad V = BH$$
$$\doteq \frac{3(2)}{2} \qquad \doteq 3(16)$$
$$\doteq 3 \qquad \doteq 48$$

The answer is reasonable.

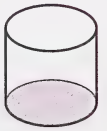
Example 2

Jonathon is measuring the dimensions of a cylindrical bottle. The diameter of the base is 6 cm. The height of the bottle is 8 cm. What is the volume of the bottle? **Note:** Round the answer to the nearest tenth.

Solution

Step 1: Calculate the area of the base (B). **Note:** The base is a circle with a diameter of 6 cm.

$$B = \pi r^2$$
$$\doteq 3.14(3)^2 \qquad d = 6 \text{ cm}, \; r = 3 \text{ cm}$$
$$\doteq 3.14(9) \quad \leftarrow \text{Use a calculator.}$$
$$\doteq 28.26$$



The area of the base (B) is about 28.26 cm^2.

Step 2: Calculate the volume (V) of the cylinder.

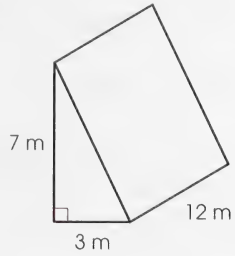
$$V = BH$$
$$\doteq 28.26(8) \quad \leftarrow \text{Use a calculator.}$$
$$\doteq 226.08 \quad \leftarrow \text{Round to nearest tenth.}$$
$$\doteq 226.1$$

The volume (V) of the bottle is about 226.1 cm^3.

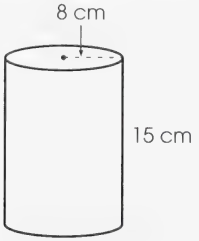
Note: Use estimation to determine if the answer is reasonable.

3. Calculate the volume of each of the following objects. Round the answers to the nearest tenth. **Note:** Estimate to determine if your answers are reasonable.

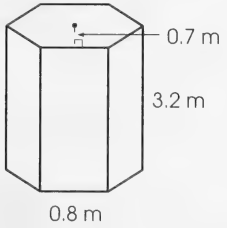
a.



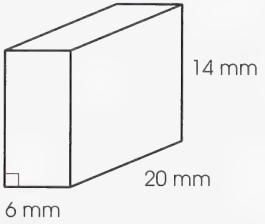
b.



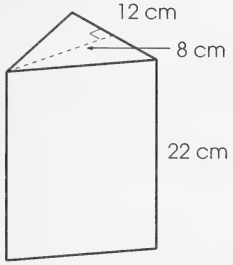
c.



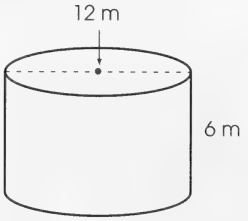
d.



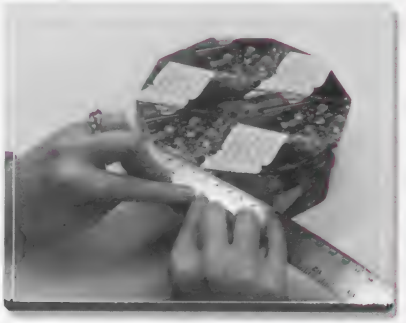
e.



f.



4. This box is a prism. The base of the prism is a decagon. Each side of the base is 5.0 cm. The apothem of the base is 7.4 cm. The height of the box is 5.8 cm. What is the volume of the box?

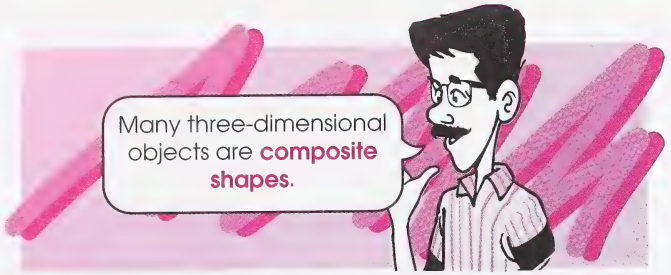


5. A Canadian $1 coin (a loonie) is a prism. The base of the coin has 11 sides of 8.0 mm. The apothem of the base is 13.3 mm. The height of the coin is 2.0 mm. What is the volume of a loonie? **Note:** Round the answer to the nearest tenth.

6. How much wax is needed to make a cylindrical candle with a radius of 5 cm and a height of 12 cm? **Note:** Round the answer to the nearest tenth.

Check your answers by turning to the Appendix.

Volume of a Composite Shape

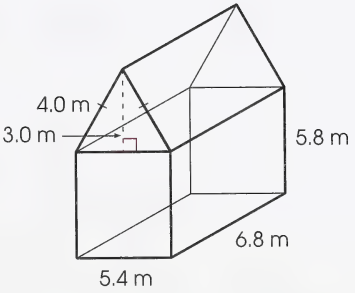


A composite shape is a combination of two or more basic shapes.

You can find the volume of composite shapes made up of prisms and cylinders.

Example 1

A metal storage shed has the dimensions shown in the diagram. Find the volume of the shed.
Note: Round the answer to the nearest tenth.

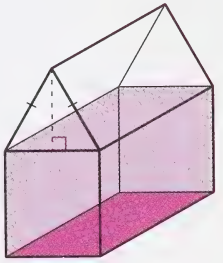


Solution

Step 1: Determine what shapes make up the shed.

There is a rectangular prism and a triangular prism.

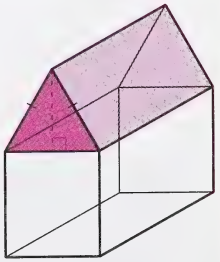
Step 2: Find the area of the base of the rectangular-prism portion of the shed. Then calculate the volume of the rectangular-prism portion.



$$B = \ell w \qquad V = BH$$
$$= 6.8(5.4) \qquad = 36.72(5.8)$$
$$= 36.72 \qquad = 212.976$$
$$\doteq 213.0$$

This portion has a volume of about 213.0 m^3.

Step 3: Find the area of the base of the triangular-prism portion of the shed. Then calculate the volume of the triangular-prism portion.



$$B = \frac{bh}{2} \qquad V = BH$$
$$= \frac{5.4(3.0)}{2} \qquad = 8.1(6.8)$$
$$= 55.08$$
$$= 8.1 \qquad \doteq 55.1$$

This portion has a volume of about 55.1 m^3.

Step 4: Find the total volume of the shed.

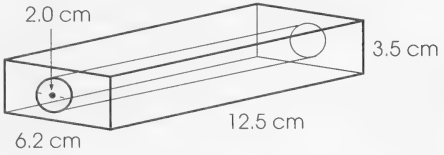
$$213.0 + 55.1 \doteq 268.1$$

The total volume of the shed is about 268.1 m^3.

Note: Use estimation to determine if the answer is reasonable.

Example 2

What is the volume of metal in this block? (A hole goes all the way through the block.) **Note:** Round the answer to the nearest tenth.



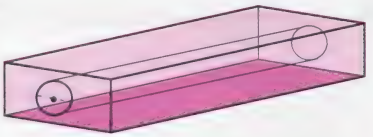
If the diameter of the cylinder is 2.0 cm, the radius is 1.0 cm.

Solution

Step 1: Determine what shapes make up the block.

There is a rectangular prism with a cylinder removed.

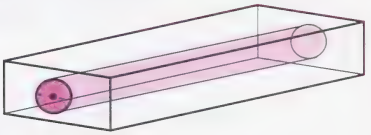
Step 2: Find the area of the base of the rectangular-prism portion of the block. Then calculate the volume of the rectangular-prism portion.



$$B = \ell w \qquad V = BH$$
$$= 12.5(6.2) \qquad = 77.5(3.5)$$
$$= 77.5 \qquad = 271.25$$
$$\doteq 271.3$$

The volume of this portion is about 271.3 cm^3.

Step 3: Find the area of the base of the cylindrical portion of the block. Then calculate the volume of the cylindrical portion.



$$B = \pi r^2 \qquad V = BH$$
$$\doteq 3.14(1.0)^2 \qquad \doteq 3.14(12.5)$$
$$\doteq 3.14(1) \qquad \doteq 39.25$$
$$\doteq 3.14 \qquad \doteq 39.3$$

The volume of this portion is about 39.3 cm^3.

Step 4: Calculate the volume of metal in the block.

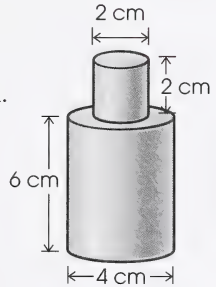
$$271.3 - 39.3 \doteq 232.0$$

There are about 232.0 cm^3 of metal in the block.

Note: Use estimation to determine if the answer is reasonable.

Use a calculator to answer questions 7 to 9.

7. What is the volume of this bottle?
Note: Round the answer to the nearest tenth.

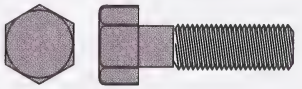


8. This lunch box has a curved top which is one-half of a cylinder. What is the volume of the lunch box? **Note:** Round the answer to the nearest tenth.

9. a. To form a hex nut, a circular hole is cut out of a piece of metal that is in the shape of a hexagonal prism.

In this hex nut, the hole has a diameter of 5 mm. The hexagonal prism is 6 mm high. Each side of the hexagonal base is 7 mm and the apothem is 7 mm. What is the volume of this hex nut? **Note:** Round the answer to nearest tenth.

b. A hex-head bolt is a piece of metal in the shape of a hexagonal prism and a cylinder



In this hex-head bolt the diameter of the cylinder portion is 1.5 cm and the height is 5.0 cm. The hexagonal-prism part is 1.0 cm high. Each side of the hexagonal base is 0.8 cm and the apothem is 0.8 cm. What is the volume of the hex-head bolt? **Note:** Round the answer to the nearest tenth. Do not consider the threads on the cylinder part.

Check your answers by turning to the Appendix.

Consumer Packaging

In order to make shipping easier, some items are packaged in boxes that are rectangular prisms.

10. With its arms down, this teddy bear is 16 cm wide and 12 cm deep. It is 35 cm high.

a. What are the minimum dimensions of the box required to package the teddy bear?

b. What is the minimum volume of the box required to package the teddy bear?

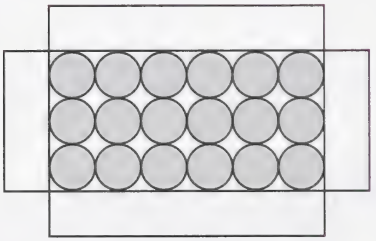
11. This rocking chair is 1.0 m high, 0.6 m wide, and 0.7 m deep.

a. What are the minimum dimensions of the box required to ship the rocking chair?

b. What is the minimum volume of the box required to package the rocking chair?

12. A manufacturer packages 18 cans of soup in a cardboard carton as shown. The cardboard carton is 36 cm × 18 cm × 11 cm.

The expression 36 cm × 18 cm × 11 cm is read as, "36 cm by 18 cm by 11 cm."



Top view of carton

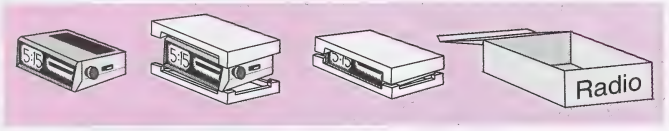
If each can has a diameter of 6 cm and is 11 cm high, how much empty space is in the carton?

Check your answers by turning to the Appendix.

In order to ship a fragile item such as a computer more safely, it is first packed in a styrofoam jacket. Then, the item and the styrofoam jacket are packed in a box.

Packing material such as styrofoam chips may also be used to cushion a fragile item packed in a box.

13. A radio is placed in a styrofoam jacket and packed in a box.

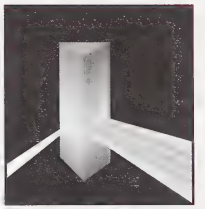


a. If the radio is $26\ \text{cm} \times 16\ \text{cm} \times 13\ \text{cm}$, what is its volume?

b. If the box is $30\ \text{cm} \times 20\ \text{cm} \times 16\ \text{cm}$, what is its volume?

c. What is the approximate volume of the styrofoam used to package this radio?

14. A scientific-supply company packs a glass prism in a box. Styrofoam chips are used as packing material.

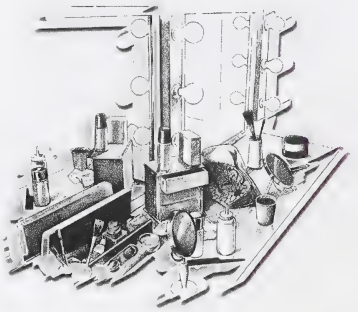


a. If the box in which the prism is packed is $100\ \text{mm} \times 50\ \text{mm} \times 50\ \text{mm}$, what is the volume of the box?

b. The base of the prism is an equilateral triangle with sides of 25 mm and a height of 22 mm. The height of the prism is 75 mm. What is its volume?

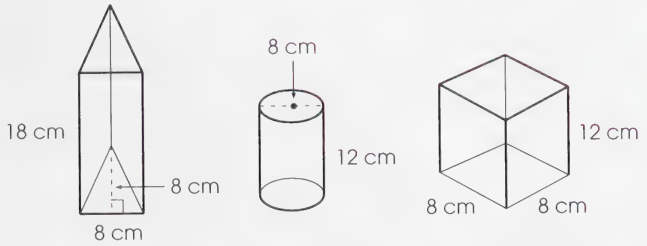
c. What is the volume of the styrofoam chips?

Check your answers by turning to the Appendix.



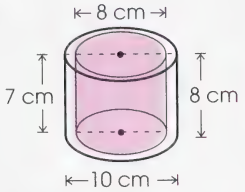
Manufacturers carefully choose the shape of the containers that hold products such as shampoo, perfume, and facial cream.

15. Look at the following containers.



a. Which container has the most unusual shape?
b. Which container do you think is easiest to handle?
c. Which container has the greatest volume?

16. Beauty Hands Inc. sells its facial cream in a jar like this. The outside of the jar has a diameter of 10 cm and a height of 8 cm. The walls of the jar are very thick. Consequently, the inside of the jar has a diameter of 8 cm and a height of 7 cm.



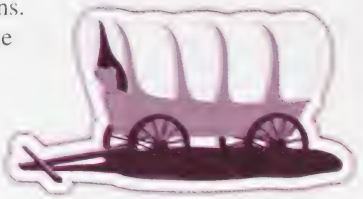
a. What is the volume of the jar?
b. What is the volume of the space for facial cream in the jar?
c. Why do you think the walls of the jar are very thick?

Check your answers by turning to the Appendix.

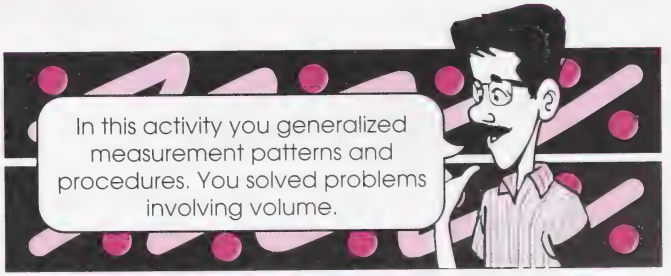
Now Try This

Use a problem-solving strategy to answer the following question.

17. A wagon train had 96 wagons. Each wagon carried the same number of people. When 12 wagons broke down, each of the other wagons had to carry one more person. How many people were in each wagon originally?

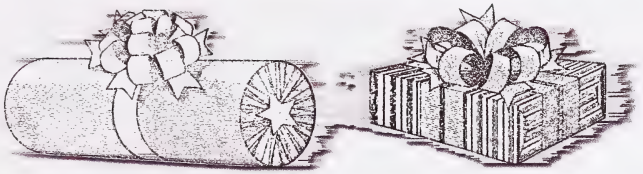


Check your answer by turning to the Appendix.



Activity 2: Surface Area

Estimating the amount of wrapping paper needed to gift-wrap a present can sometimes be a problem. Which of the following presents do you think required more wrapping paper?



The amount of wrapping paper is related to the surface area of the container being wrapped.

In this activity you will calculate the surface area of prisms, cylinders, and composite shapes.

View the program *Areas (of Circles and Cylinders)* from the series *Mathways*. It reviews how to calculate the area of a circle and introduces how to find the surface area of a cylinder.

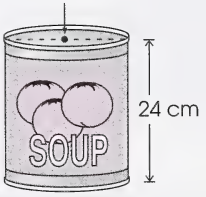
1. Explain how the surface area of a can of tennis balls was calculated in the video. **Note:** Give the method, but don't perform the calculations.

Check your answer by turning to the Appendix.

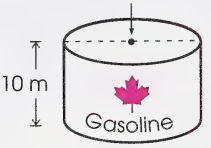
Use the method used in the video to answer questions 2 and 3.

2. Find the surface area of the following cylinders. **Note:** Round each answer to the nearest tenth.

a.



b.



3. A 355-mL can of pop approximates a cylinder. If a can of pop has a diameter of about 6 cm and a height of 12 cm, what is its surface area? **Note:** Round the answer to the nearest tenth.

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Check your answers by turning to the Appendix.

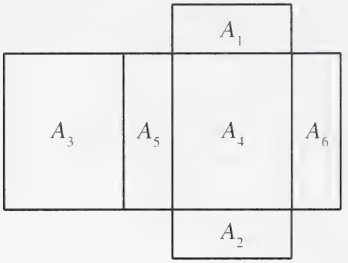
Example 1

Find the surface area of this cereal box.

Solution

Step 1: Use this reasoning.

The surface area of the box is the sum of these 6 areas.
Note: $A_1 = A_2$, $A_3 = A_4$, $A_5 = A_6$



Step 2: Calculate the area of A_1 and A_2. **Note:** Each rectangle is 19 cm × 5 cm.

$$A_1 = \ell w \qquad \therefore\ A_2 = 95$$
$$= 19(5) \qquad \text{and}$$
$$= 95 \qquad A_1 + A_2 = 190$$

The area of A_1 and A_2 together is 190 cm^2.

Step 3: Calculate the area of A_3 and A_4. **Note:** Each rectangle is 25 cm × 19 cm.

$$A_3 = \ell w \qquad \therefore\ A_4 = 475$$
$$= 25(19) \qquad \text{and}$$
$$= 475 \qquad A_3 + A_4 = 950$$

The area of A_3 and A_4 together is 950 cm^2.

Step 4: Calculate the area of A_5 and A_6. **Note:** Each rectangle is 25 cm × 5 cm.

$$A_5 = \ell w \qquad \therefore\ A_6 = 125$$
$$= 25(5) \qquad \text{and}$$
$$= 125 \qquad A_5 + A_6 = 250$$

The area of A_5 and A_6 together is 250 cm^2.

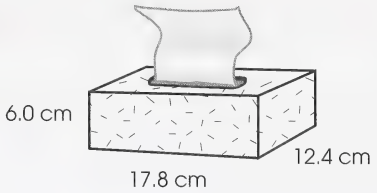
Step 5: Find the surface area of the box.

$$(A_1 + A_2) + (A_3 + A_4) + (A_5 + A_6) = 190 + 950 + 250$$
$$= 1390$$

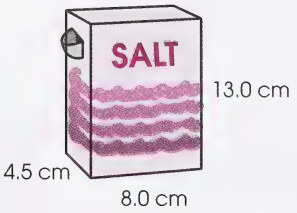
The surface area of the box is 1390 cm^2.

4. Find the surface area of each of the following containers.

a.



b.



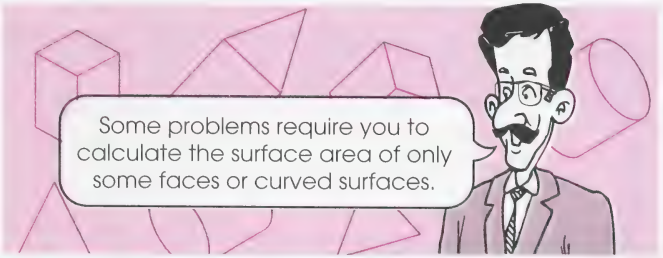
5. This box is a prism. The base of the box is a hexagon. Each side of the base is 7.5 cm. The apothem of the base is 8.0 cm. The height of the box is 5.8 cm. What is the surface area of the box?

6. This chocolate box is a triangular prism. The height of the box is 16.7 cm. The triangular base has sides of 3.0 cm and a height of 2.5 cm.

What is the surface area of the box? (You have already investigated the volume of this box.) **Note:** Round the answer to the nearest tenth.

[1]

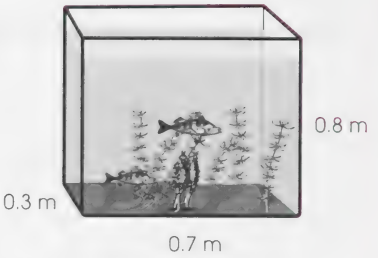
Check your answers by turning to the Appendix.



[1] © Kraft Foods Canada, Inc. All rights reserved.

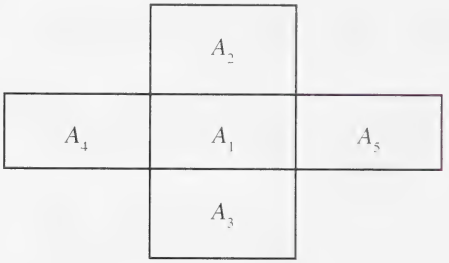
Example 2

What is the surface area of the exterior of this aquarium?
Hint: Express the surface area in m^2.



Solution

Step 1: Use this reasoning. The surface area of the exterior of the aquarium is the sum of these 5 areas. **Note:** $A_2 = A_3$; $A_4 = A_5$.



Step 2: Calculate the area of A_1.
Note: The rectangle is 0.7 m × 0.3 m.

$$A_1 = \ell w$$
$$= 0.7(0.3)$$
$$= 0.21$$

The area of A_1 is 0.21 m^2.

Step 3: Calculate the area of A_2 and A_3.
Note: Each rectangle is 0.7 m × 0.8 m.

$$A_2 = \ell w \qquad \therefore\ A_3 = 0.56$$
$$= 0.7(0.8) \qquad \text{and}$$
$$= 0.56 \qquad A_2 + A_3 = 1.12$$

The area of A_2 and A_3 together is 1.12 m^2.

Step 4: Calculate the area of A_4 and A_5.
Note: Each rectangle is 0.8 m × 0.3 m.

$$A_4 = \ell w \qquad \therefore\ A_5 = 0.24$$
$$= 0.8(0.3) \qquad \text{and}$$
$$= 0.24 \qquad A_4 + A_5 = 0.48$$

The area of A_4 and A_5 together is 0.48 m^2.

Step 5: Find the surface area of the exterior of the aquarium.

$$A_1 + (A_2 + A_3) + (A_4 + A_5) = 0.21 + 1.12 + 0.48$$
$$= 1.81$$

The surface area of the exterior of the aquarium is 1.81 m^2.

7. A cylindrical pool has a plastic liner covering its interior walls and floor. If the pool has a diameter of 4 m and a depth of 2 m, what is the surface area of the plastic liner?

8. A cake has a diameter of 20 cm and a height of 13 cm.

a. If the top and sides of the cake are iced, what is the area of the surfaces of the cake that are iced?

b. The cake is cut into two identical layers. If the top and sides of each layer are iced, what is the area of the iced surfaces of the cake?

9. A rectangular room is 8 m long and 4 m wide. The walls are 3 m high.

If 1 L of paint covers 12 m^2, how much paint is required to paint the ceiling and walls of the room?

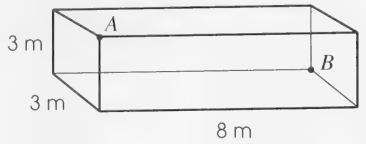
Hint: Do not consider windows and doors.

Check your answers by turning to the Appendix.

Now Try This

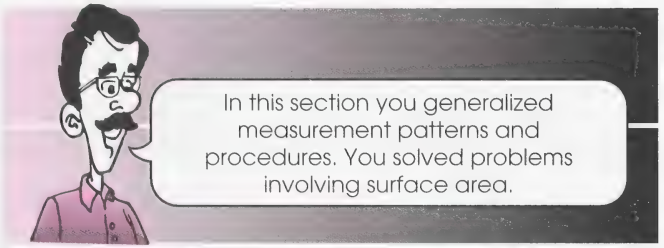
Use a problem-solving strategy to answer the following question.

10. A spider is at vertex A in a rectangular room. The spider wants to crawl to vertex B.



What is the shortest path the spider can take to get from vertex A to vertex B? What is the length of the path?

Check your answer by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts, it is recommended that you do the Enrichment. You may decide to do both.

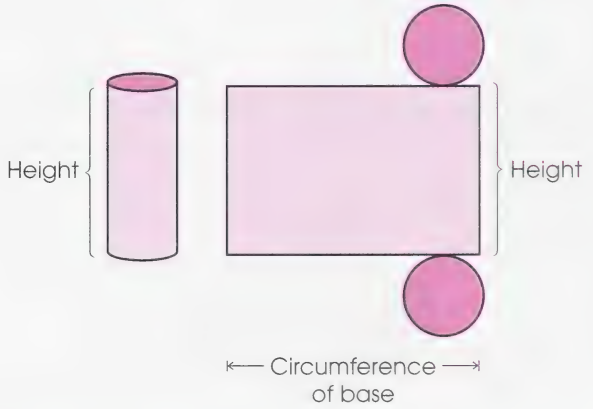
Extra Help

In this section you developed a single formula to calculate the volume of any cylinder or prism.

$$V = BH$$

A single formula can be developed to calculate the surface area of any cylinder.

Look at this net of a cylinder.



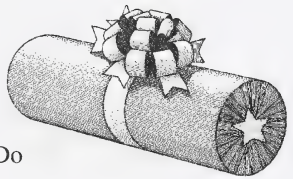
The net is made up of two bases (which are congruent circles) and one rectangle (which is formed when the curved surface of the cylinder is flattened). The width of the rectangle is the height of the cylinder. The length of the rectangle is equal to the circumference of the base of the cylinder.

Therefore, to calculate the surface area of any cylinder, you can use the following formula, where SA is the surface area of the cylinder, B is the area of each base of the cylinder, C is the circumference of the base of the cylinder, and H is the height of the cylinder.

$$SA = 2B + CH$$

Example 1

A gift is placed in a cylinder. The cylinder has a diameter of 10 cm and is 25 cm high. Then the cylinder is gift-wrapped. What is the surface area of the cylinder which will be wrapped? **Note:** Do not consider overlapping.



Solution

Step 1: Calculate the area of each base. **Note:** Each base is a circle with a diameter of 10 cm.

$$B = \pi r^2$$
$$\doteq 3.14\left(5^2\right)$$
$$\doteq 3.14(25)$$
$$\doteq 78.5$$

$d = 10$ cm
$r = 5$ cm

The area of each base is about 78.5 cm^2.

Step 2: Calculate the circumference of the base.

$$C = \pi d$$
$$\doteq 3.14(10)$$
$$\doteq 31.4$$

The circumference of the base is about 31.4 cm.

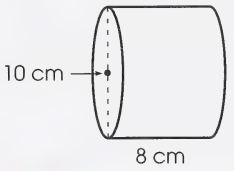
Step 3: Find the surface area of the cylinder.

$$SA = 2B + CH$$
$$\doteq 2(78.5) + 31.4(25)$$
$$\doteq 157 + 785$$
$$\doteq 942$$

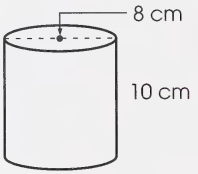
The surface area of the cylinder is about 942 cm^2.

1. Use the formula $SA = 2B + CH$ to calculate the surface area of each of the following solid cylinders. **Note:** Round each answer to the nearest whole number.

a.



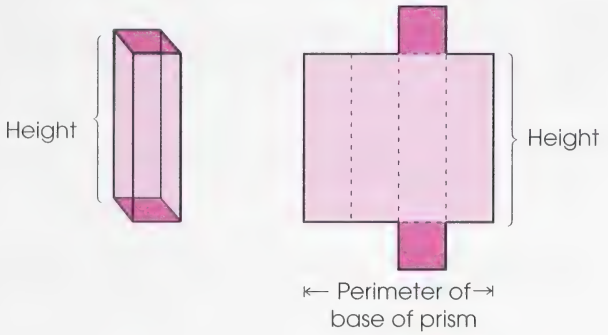
b.

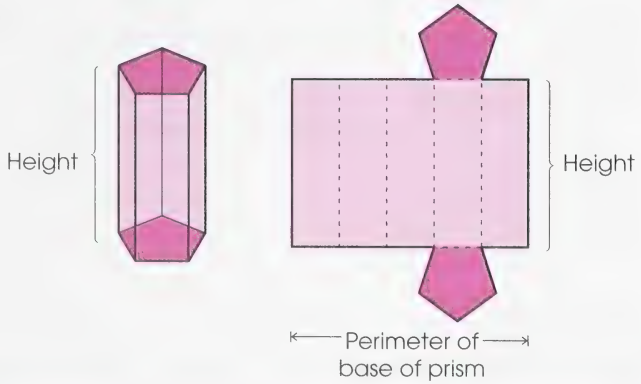


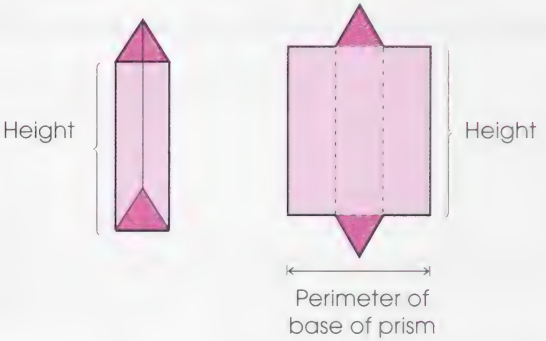
Check your answers by turning to the Appendix.

A single formula can be developed to calculate the surface area of any prism.

Look at the nets of these prisms. Each net is made up of the two bases (which are congruent polygons) and a large rectangle (made up of several congruent rectangles). The width of each large rectangle is equal to the height of the prism. The length of each large rectangle is equal to the perimeter of the base of the prism.





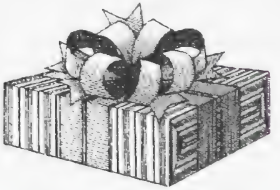


Therefore, to calculate the surface area of any prism, you can use the following formula, where SA is the surface area of the prism, B is the area of each base of the prism, P is the perimeter of the base of the prism, and H is the height of the prism.

$$SA = 2B + PH$$

Example 2

A gift is placed into a box that is a rectangular prism. The rectangular prism is 20 cm long, 12 cm deep, and 8 cm high. What is the surface area of the box which will be wrapped? **Note:** Do not consider overlapping.



Solution

Step 1: Calculate the area of the base. **Note:** The base is a rectangle that is 20 cm × 12 cm.

$$\begin{aligned} B &= \ell w \\ &= 20(12) \\ &= 240 \end{aligned}$$

The area of the base is 240 cm^2.

Step 2: Calculate the perimeter of the base. **Note:** The base is a rectangle 20 cm × 12 cm.

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2(20) + 2(12) \\ &= 40 + 24 \\ &= 64 \end{aligned}$$

The perimeter of the base is 64 cm.

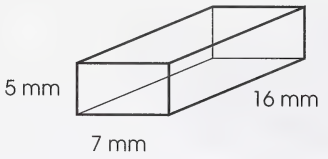
Step 3: Calculate the surface area of the box.

$$\begin{aligned} SA &= 2B + PH \\ &= 2(240) + 64(8) \\ &= 480 + 512 \\ &= 992 \end{aligned}$$

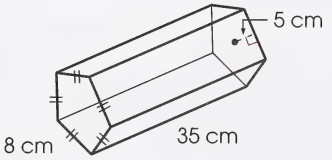
The surface area of the box is 992 cm^2.

2. Use the formula $SA = 2B + PH$ to calculate the surface area of each of the following prisms.

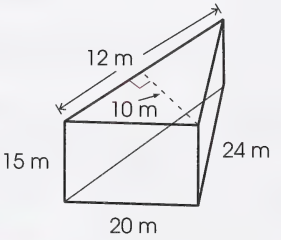
a.



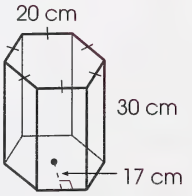
b.



c.

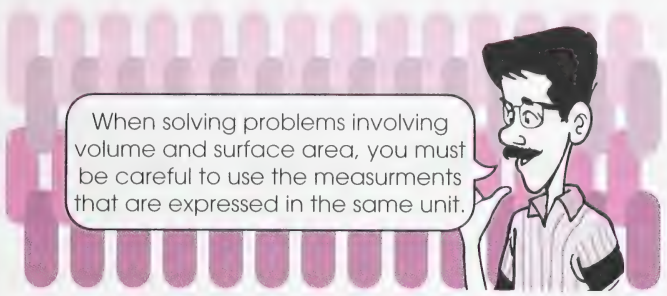


d.



Check your answers by turning to the Appendix.

Enrichment

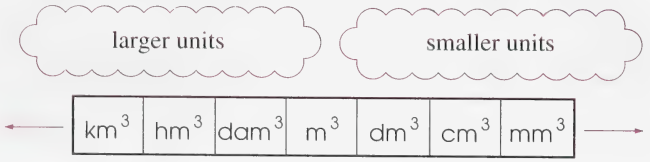

When solving problems involving volume and surface area, you must be careful to use the measurments that are expressed in the same unit.

In the Enrichment for Section 2 of Module 4, you changed from one square unit to another. In this activity you will change from one cubic unit to another.

It is easy to change from one cubic unit to another in the metric system because the cubic units are multiples of 1000.

$1\ km^3 = 1000\ hm^3$ $\quad 1\ hm^3 = 1000\ dam^3$ $\quad 1\ dam^3 = 1000\ m^3$
$1\ m^3 = 1000\ dm^3$ $\quad 1\ dm^3 = 1000\ cm^3$ $\quad 1\ cm^3 = 1000\ mm^3$

You can use the following metric ladder to help you remember the metric units of volume.

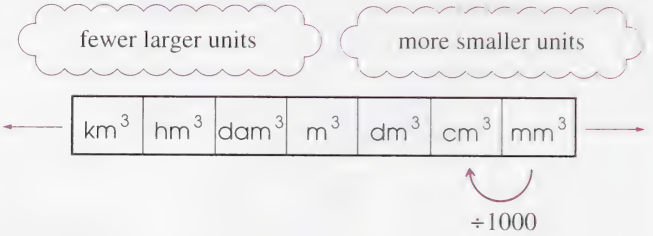


Example 1

Change $1500\ mm^3$ to cubic centimetres.

Solution

Use the metric ladder.



To change from cubic millimetres to cubic centimetres, divide by 1000. (Dividing by 1000 moves the decimal point three places to the left.)

$1500\ mm^3 = 1.5\ cm^3$

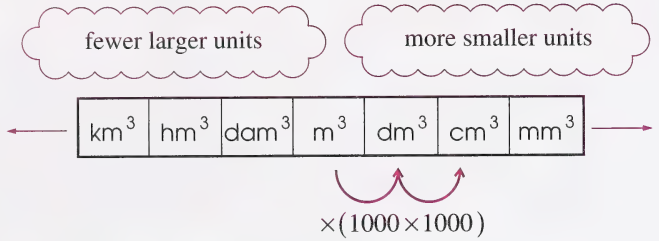
Note: Because you are changing to larger units, there will be fewer cubic centimetres than cubic millimetres.

Example 2

Change 8 m^3 to cubic centimetres.

Solution

Use the metric ladder.



To change from cubic metres to cubic centimetres, multiply by 1 000 000. (Multiplying by 1 000 000 moves the decimal point six places to the right.)

$$8\ m^3 = 8\,000\,000\ cm^3$$

Note: Because you are changing to smaller units, there will be more cubic centimetres than cubic metres.

1. Change each of the following measurements to cubic centimetres.

a. 3 m^3 **b.** 7800 mm^3

c. 1.82 m^3 **d.** 450 000 mm^3

2. Change each of the following measurements to cubic metres.

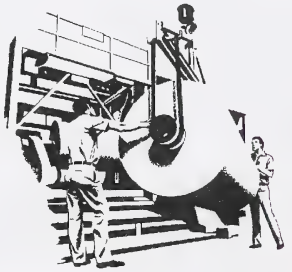
a. 0.5 km^3 **b.** 75 000 000 cm^3

c. 1.2 km^3 **d.** 592 000 mm^3

Check your answers by turning to the Appendix.

You are now ready to solve problems involving change of units.

3. Newsprint is shipped in large cylindrical rolls. The rolls have a diameter of 102 cm and a length of 137 cm.



Note: When answering the following questions, do not consider the cylinder about which the paper is rolled. Round each answer to the nearest hundredth.

a. What is the volume (in cm^3) of a roll of newsprint?

b. What is the volume (in m^3) of a roll of newsprint?

4. A jewellery box is $90\ mm \times 80\ mm \times 75\ mm$.

   a. What is the volume (in mm^3) of the box?

   b. What is the volume (in cm^3) of the box?

Check your answers by turning to the Appendix.

Did You Know?

Every winter it becomes very important to know when the ice on a pond or lake is safe to use. The thickness of the ice on a pond can be determined by drilling holes with an ice auger.

In the following chart are some guidelines for determining the safety of freshwater ice.

Required Minimum Ice Thickness in Centimetres	Description of Safe Moving Load
5	one person on skis
6	one person on foot or skates
8	one snowmobile
18	one passenger automobile

5. During the winter, Marion likes to snowmobile on a nearby pond. If the pond has an area of $5\ km^2$ and is covered by a sheet of ice that is 8 cm thick, what is the volume (in m^3) of the ice?

Check your answers by turning to the Appendix.

Do you think it is possible for ice to support very heavy machinery?

In the Appendix find and read the article "Offshore Drilling Platforms of Floating Ice."

INTERNET

Offshore drilling is occurring today in Hibernia, off the coast of Newfoundland. Use the Internet to discover what the offshore drilling platforms are like today. You may find the following site useful:

http://www.hibernia.ca/hmdc/default.htm

Conclusion

PHOTO SEARCH LTD.

In this section you generalized measurement patterns and procedures and solved problems involving volume and surface area.

The casserole dish in the photograph is cylindrical. What do you think the volume of this dish is? What do you think the surface area of the dish is?

Did you know that the amount of surface area will determine how fast the casserole will be heated through? The greater the surface area, the faster the food will be heated.

Assignment

Assignment Booklet

You are now ready to complete the assignment for Section 2.

Module Summary

PHOTO SEARCH LTD

In this module you investigated three-dimensional space. You constructed many 3-D shapes and investigated their properties. You generalized measurement patterns and procedures and solved problems involving volume and surface area.

Manufacturers use many different shapes of containers to store food products. For example, peanut butter is stored in cylindrical containers. Macaroni-and-cheese mixes are stored in boxes that are rectangular prisms.

Which shapes of containers fit together on a shelf with the least amount of wasted space?

Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment.

You are also ready to complete the Course Survey for Mathematics 8. The survey follows the Course Summary.

Course Summary

Mathematics 8

Congratulations on completing the six modules of Mathematics 8. Now that all your assignments have been completed and reviewed, it's time for the last step in the Mathematics 8 course—the Final Test. With all the hard work and time you put into the course, you are sure to do well! Good Luck!

COURSE SURVEY FOR MATHEMATICS 8 (1997)

After you have completed the assignments in this course, please fill out this questionnaire and mail it to the address given on the last page. This course is designed in a new distance learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.

Name _____ Age ❑ under 19 ❑ 19 to 40 ❑ over 40

Address _____ File No. _____

_____ Date _____

_____

Design

1. This course contains a series of Student Module Booklets. Do you like the idea of separate booklets?

_____

_____

2. Have you ever enrolled in a correspondence or distance learning course that arrived as one large volume?

❑ Yes ❑ No If yes, which style do you prefer?

_____

_____

3. The Student Module Booklets contain a variety of self-assessed activities. Did you find it helpful to be able to check your work and have immediate feedback?

❑ Yes ❑ No If yes, explain.

_____

_____

4. Were the questions and directions easy to understand?

❑ Yes ❑ No If no, explain.

_____

_____

5. Each section contains follow-up activities. Which type of follow-up activity did you choose?

   ❑ mainly extra help
   ❑ mainly enrichment
   ❑ a variety
   ❑ none

   Did you find these activities beneficial?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

6. Did you understand what was expected in the Assignment Booklets?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

7. The course materials were designed to be completed by students working independently at a distance. Were you always aware of what you had to do?

   ❑ Yes ❑ No If no, provide details.

   _____

   _____

8. This distance learning course may include an assortment of drawings, photographs, and charts.

   a. Did you find the visuals in this course helpful?

      ❑ Yes ❑ No Comment on the lines below.

      _____

      _____

   b. Did you find the variety of visuals in this course motivating?

      ❑ Yes ❑ No Comment on the lines below.

      _____

      _____

9. Suggestions for audiocassette, videocassette, and computer activities may have been included in the course. Did you complete these media activities?

   ❑ Yes ❑ No Comment on the lines below.

   _____

   _____

Only students enrolled in a Junior High course need to complete the following question.

10. The Student Module Booklet may have directed you to work with your learning facilitator. How well did you work as a team?

    Student's comments: _____

    _____

    _____

    _____

    Learning Facilitator's comments: _____

    _____

    _____

    _____

Course Content

1. Was enough detailed information provided to help you learn the expected skills and ojectives?

   ❑ Yes ❑ No Comment on the lines below.

   _____

   _____

2. Did you find the work load reasonable?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

3. Did you have any difficulty with the reading level?

❑ Yes ❑ No Please comment.

_____

_____

4. How would you assess your general reading level?

❑ poor reader ❑ average reader ❑ good reader

5. Was the material presented clearly and with sufficient depth?

❑ Yes ❑ No If no, explain.

_____

_____

General

1. What did you like least about the course?

_____

_____

2. What did you like most about the course?

_____

_____

Additional Comments

_____

_____

_____

_____

_____

_____

_____

Only students enrolled with the Alberta Distance Learning Centre need to complete the remaining questions.

1. Did you contact the Alberta Distance Learning Centre for help or information while doing your course?

   ❑ Yes ❑ No If yes, approximately how many times? _____

   Did you find the staff helpful?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

2. Were you able to fax any of your assignment response pages?

   ❑ Yes ❑ No If yes, comment on the value of being able to do this.

   _____

   _____

3. If you mailed your assignment response pages, how long did it take for their return?

   _____

   _____

4. Was the feedback you received from your correspondence or distance learning teacher helpful?

   ❑ Yes ❑ No Please comment.

   _____

   _____

   _____

   _____

Thanks for taking the time to complete this questionnaire. Your feedback is important to us. Please return this questionnaire to the address on the right.

If you are enrolled at the Alberta Distance Learning Centre and have been mailing your Assignment Booklets to ADLC, you may return this questionnaire with the final Assignment Booklet in the course.

Instructional Design and Development
Learning Technologies Branch
Box 4000
Barrhead, Alberta
T7N 1P4

APPENDIX

- Problem-Solving Strategies
- Glossary
- Suggested Answers
- Articles/Puzzles/Learning Aids

Problem-Solving Strategies

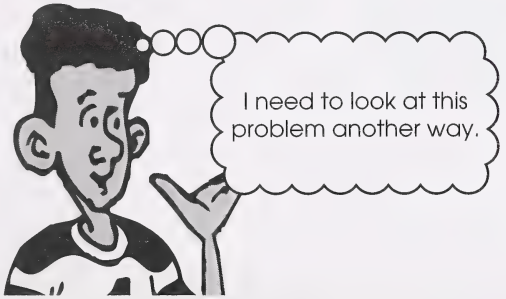
Be sure to study the following problem-solving strategies and use them when you encounter problems in this course.

Whichever method of problem solving you may choose to use, you should do each of the following:

- Show that you understand the problem by showing all the steps needed for finding the answer.
- Draw and neatly label any diagrams, graphs, or charts that may help you answer the problem.
- Write a concluding sentence that answers the question being asked.

Changing Your Point of View

Sometimes you find that you cannot solve a particular problem because of your "mind set." Perhaps you made an assumption about the problem that is incorrect. If your attempts to solve a problem are not being successful, it is often helpful to try to change your point of view.

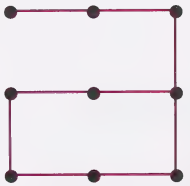


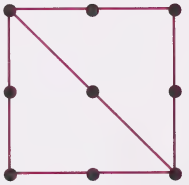
Example

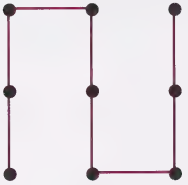
Draw four line segments through the following nine dots without lifting your pencil from the paper and without retracing your path.

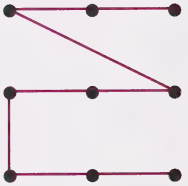
Solution

Step 1: You make several attempts at solving the problem.


5 line segments

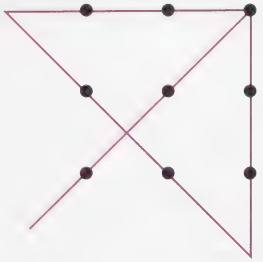

5 line segments


5 line segments


5 line segments

Step 2: You realize you are blocked in your attempts. You have made the assumption that the dots must be connected in a way similar to a child's dot-to-dot picture.

In order to solve the problem you must change your point of view and realize that the line segments may lie outside the confines of the dots.



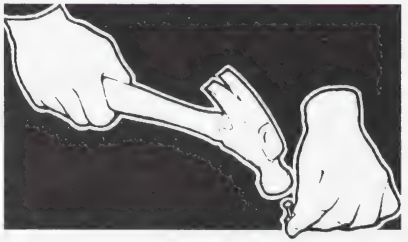
4 line segments

Using Objects

Objects can be used to help you solve a problem.

Example 1

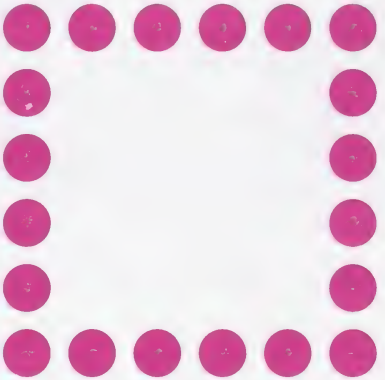
To enclose a square lot with a fence, 20 posts are used. If the posts are placed 3 m apart, what is the length of one side of the lot?



Solution

Step 1: Use objects such as blocks, checkers, or coins to represent the posts.

The advantage of using objects to represent the posts is that you can easily rearrange them.



Step 2: Count the number of spaces between the posts on one side of the square.

There are five spaces between the six posts on one side of the square.

Step 3: Calculate the length of one side of the square.

$$5 \times 3 = 15$$

The length of one side of the lot is 15 m.

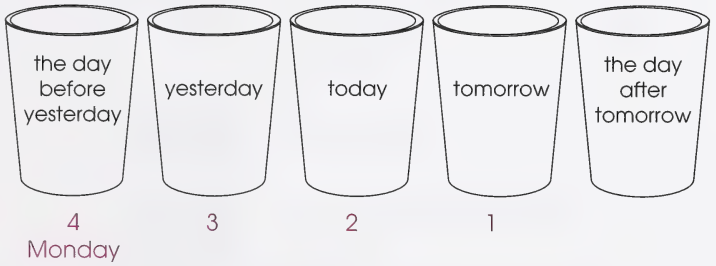
Example 2

Four days before the day after tomorrow was Monday. What day of the week is it today?

Solution

Step 1: Use paper cups to represent the days. The cups could be labelled as shown.

Step 2: Begin at the cup representing the day after tomorrow and count back four days. That day was Monday.



If Monday was the day before yesterday, yesterday was Tuesday, and today is Wednesday.

Using Diagrams

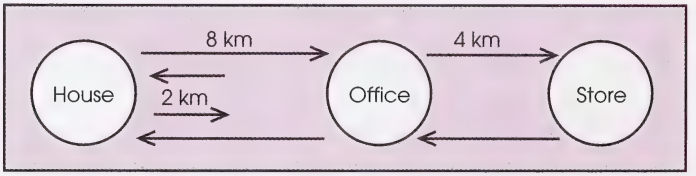
You can use sketches to solve problems.

Example 1

Chris' office is 8 km from his home. Yesterday morning he drove 2 km before he realized that he had forgotten his briefcase. He returned home to get his briefcase and then drove to his office. At noon Chris drove 4 km to a store. He then went back to the office for the rest of the afternoon. At the end of the day he drove straight home. How far did he drive yesterday?

Solution

Step 1: Draw and label a diagram to help you understand the problem.



Step 2: Calculate the distance Chris drove.

$$2+2+8+4+4+8=28$$

Chris drove 28 km.

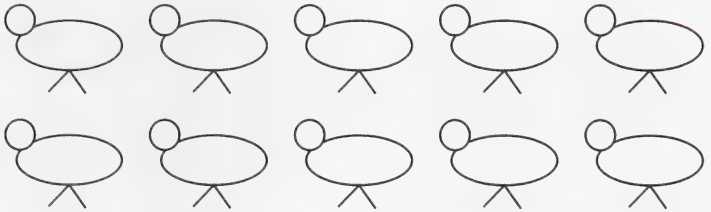
Example 2

There are ten animals in a barnyard. Some are chickens and the rest are sheep. Ruth counts 28 legs. How many chickens and how many sheep are there? **Hint:** Chickens have 2 legs and sheep have 4 legs.

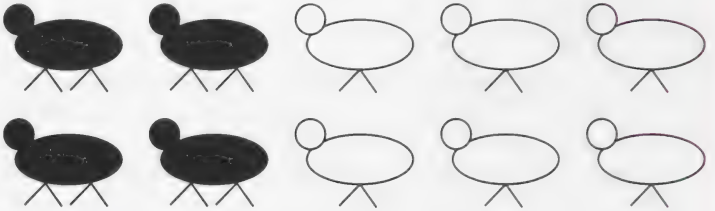
Solution

Draw a diagram to help you solve the problem.

Step 1: Every animal has at least 2 legs. So, draw a diagram of ten animals, each of which has at least 2 legs.



Step 2: The group of animals in the problem had 28 legs. So, you need to add 8 legs to the drawing. Add the legs in pairs to the sketch of each animal.



There are four sheep and six chickens.

Making an Organized List

Some problems require you to list all the possible solutions. It is important to make an organized list so that you do not miss any possibilities.

Example 1

Julio and his friends like playing a game called Bull's Eye with lawn darts. They draw a target in the dirt and assign values to the three regions of the target.

If Julio throws three darts and all three hit the target, how many different point totals are possible?

Solution

Begin with the highest possible score and list all the possibilities in order.

$9+9+9=27$ ← highest possible score
$9+9+4=22$
$9+9+3=21$
$9+4+4=17$
$9+4+3=16$
$9+3+3=15$
$4+4+4=12$
$4+4+3=11$
$4+3+3=10$
$3+3+3=9$ ← lowest possible score

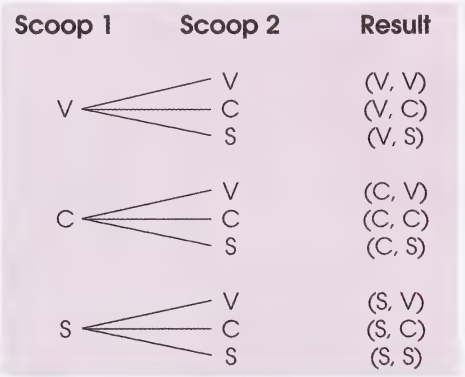
There are ten possible point totals.

Example 2

At an ice-cream parlour there are three flavours of ice cream. How many different double-scoop ice-cream cones are possible? **Hint:** A chocolate-vanilla cone is different from a vanilla-chocolate cone. Order makes a difference.

Solution

Draw a tree diagram to help you make an organized list. Use V for vanilla, C for chocolate, and S for strawberry.



Nine different double-scoop ice-cream cones are possible.

Using Venn Diagrams

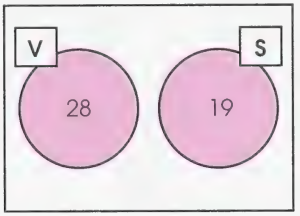
A type of diagram that is helpful in some problems is a Venn diagram. In this special kind of diagram, circles are used to represent groups of people, animals, or objects that have certain characteristics. The positioning of the circles in relation to one another represents relationships among these groups. These diagrams were named after an English mathematician named John Venn (1834–1923).

Example 1

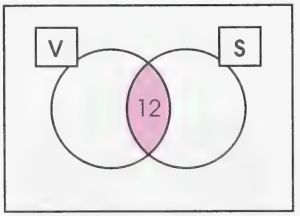
There are 28 girls on the volleyball team and 19 girls on the swim team. If 12 girls belong to both teams, how many girls belong only to the swim team?

Solution

Step 1: Represent the members of the volleyball team with a circle. Represent the members of the swim team with another circle.



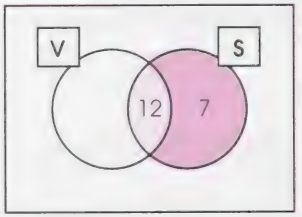
Step 2: Represent the members of both teams by the intersection of these two circles.



Step 3: Now calculate the number of girls who belong only to the swim team.

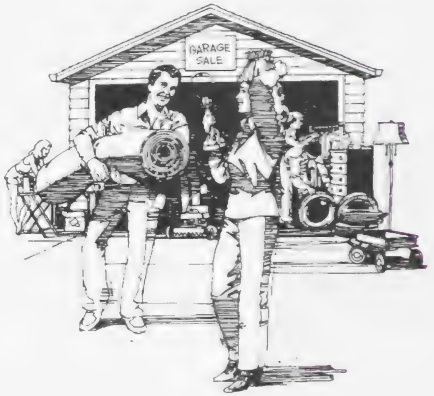
$$19 - 12 = 7$$

So, 7 girls belong only to the swim team.



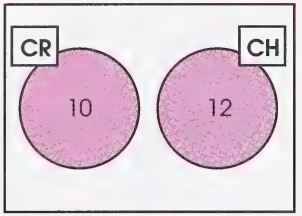
Example 2

Reilly bought some plates at a garage sale. He discovered that 10 plates were cracked, 12 plates were chipped, 6 plates were both chipped and cracked, and 2 plates were neither chipped nor cracked. How many plates did Reilly buy?

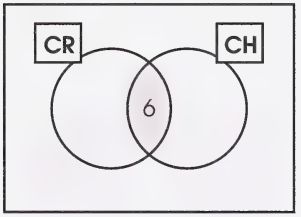


Solution

Step 1: The problem states 10 plates were cracked and 12 plates were chipped. Represent the cracked plates with a circle. Represent the chipped plates with a second circle.

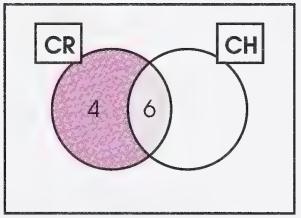


Step 2: The problem states that 6 plates were both chipped and cracked. Represent the cracked and chipped plates with the intersection of the two circles.



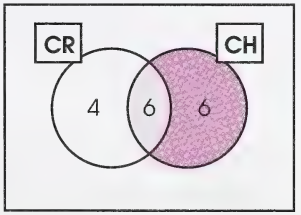
Step 3: Calculate the number of plates that were only cracked.

$$10 - 6 = 4$$

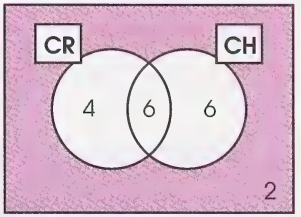


Step 4: Calculate the number of plates that were only chipped.

$$12 - 6 = 6$$



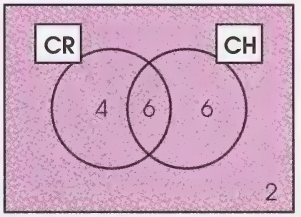
Step 5: The problem states that 2 plates were neither chipped nor cracked. Represent the undamaged plates with the region outside the circles.



Step 6: Calculate the total number of plates that were bought.

$$4 + 6 + 6 + 2 = 18$$

So, 18 plates were bought.



Example 3

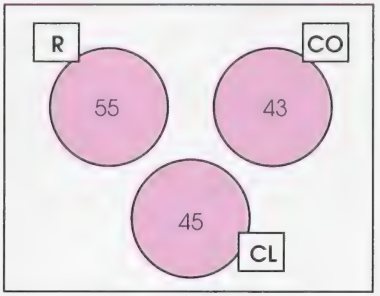
In a survey of 100 people, the following information was collected:

- 43 people like country music.
- 55 like rock music.
- 45 like classical music.
- 8 people like all three forms.
- 15 people like both country music and classical music.
- 25 like both rock and country music.
- 20 like both rock and classical music.

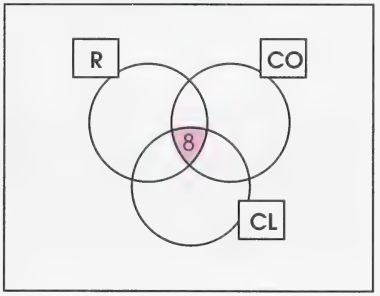
How many people do not like any of these forms of music?

Solution

Step 1: The problem states that 55 people like rock music, 43 like country music, and 45 like classical music. Use three circles to represent these preferences.

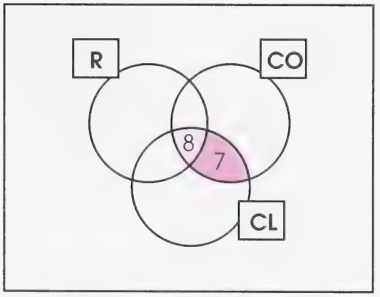


Step 2: The problem states that 8 people like all three forms. The intersection of the three circles shows the people who like all three forms.



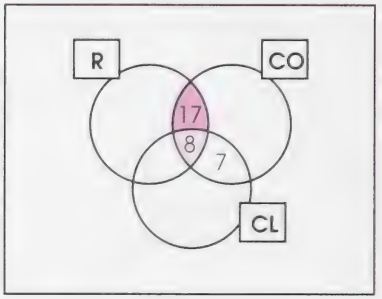
Step 3: The problem states that 15 people like both country music and classical music. Calculate the number of people who like country and classical music, but do not like rock music.

$$15 - 8 = 7$$



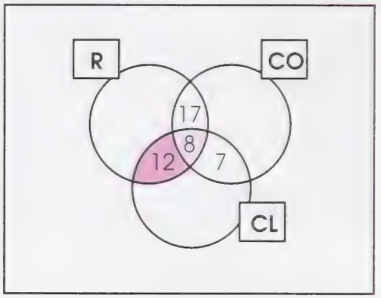
Step 4: The problem states that 25 people like both rock and country music. Calculate the number of people who like both country and rock music, but do not like classical music.

$$25 - 8 = 17$$



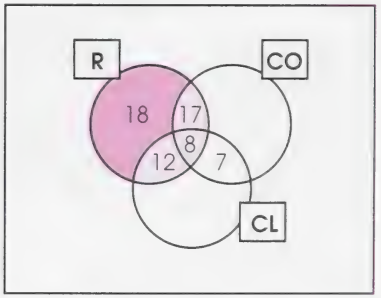
Step 5: The problem states that 20 people like both rock and classical music. Calculate the number of people who like both rock and classical music, but not country music.

$$20 - 8 = 12$$



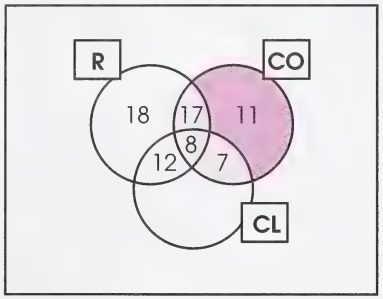
Step 6: Now that you know the number of people who like rock music and another form of music, calculate the number of people who only like rock.

$$55 - (17 + 8 + 12) = 18$$



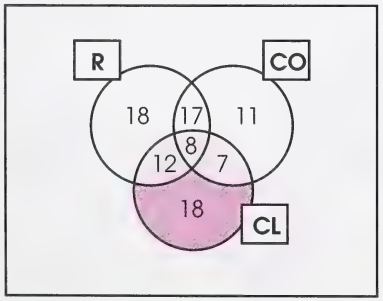
Step 7: Now that you know the number of people who like country music and another form of music, calculate the number of people who only like country music.

$$43-(17+8+7)=11$$



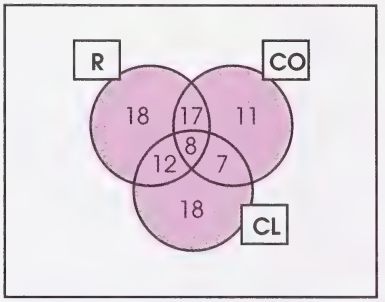
Step 8: Now that you know the number of people who like classical music and another form of music, calculate the number of people who like only classical music.

$$45-(12+8+7)=18$$



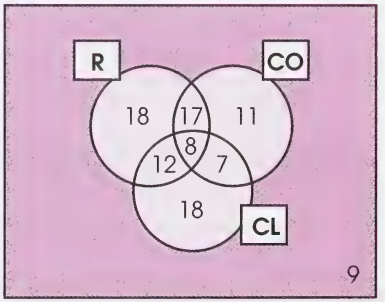
Step 9: Now you can calculate the total number of people who like rock, country, or classical music.

$$18+17+8+12+11+7+18=91$$



Step 10: The problem states that 100 people were surveyed. So, you can calculate the number of people who do not like any of these forms of music.

$$100-91=9$$



Of the 100 people surveyed, 9 people do not like any of these forms of music.

Making a Table

You can use a table to organize information.

Example

Hannah works at her mother's store after school. She earns 20¢ for each customer that she assists and 35¢ for each bag that she packs. One day she did 19 jobs and earned $4.85. How many customers did she assist that day and how many bags did she pack?

Solution

Write down all possible combinations of 19 jobs until you find a combination that totals $4.85. Use a table to organize the information.

Packing	1	2	3	4	5	6	7
Assisting	18	17	16	15	14	13	12
Earned	$3.95	$4.10	$4.25	$4.40	$4.55	$4.70	$4.85

correct guess

Hannah assisted 12 customers and packed 7 bags that day.

Guessing, Checking, and Revising

Many mathematical problems can be solved by systematic trial or guessing, checking, and revising.

Example

Shelley opened her math textbook and noticed that the product of the numbers of the two open pages was 2970. To what pages was the book opened? **Hint:** The left-hand page number of a book is even; the right-hand page number is odd.

Solution

Method 1: Using Paper and Pencil

Step 1: Note the conditions that need to be met.

- The left-hand page number must be even.
- The right-hand page number is one more than the left-hand page number.
- The product of the numbers must be 2970.

Step 2: Make a guess, check your guess, and revise if necessary.

It is helpful to organize your guesses in a table.

Left-hand page number	50	52	54
Right-hand page number	51	53	55
Product of page numbers	2550	2756	2970

correct guess

So, the book was open to pages 54 and 55.

Method 2: Using a Computer

Use a computer and a spreadsheet program to help you solve the problem.

Step 1: Label the cells on the spreadsheet. **Hint:** The cell address at the top of the spreadsheet shows which cell is active. There is also a border around the active cell. You don't type directly in this cell; instead you use the entry bar at the top of the spreadsheet.

A5 | × | ✓ | Product of page numbers

	A	B
1	Book Problem	
2		
3	Left-hand page number	
4	Right-hand page number	
5	Product of page numbers	
6		

Step 2: Enter either a number or a formula for each of the cells.

From the problem, you know that the right-hand page number is one more than the left-hand page number and that the product of the page numbers is 2970.

Here is what you enter:

- Begin with the B3 cell and enter a guess of 50.
- Next, enter = B 3 + 1 for the B4 cell. This means the number in the B4 cell will be one more than the number in the B3 cell.
- Finally, enter = B 3 * B 4 for the B5 cell. This means that the number in the B5 cell will be the product of the numbers in the B3 and B4 cells.

B5 | × | ✓ | =B3*B4

	A	B
1	Book Problem	
2		
3	Left-hand page number	50
4	Right-hand page number	51
5	Product of page numbers	2550
6		

The computer performs the calculations and the results appear in the B4 and B5 cells.

Step 3: Check your initial guess and revise it if necessary. The product is 2970, not 2550; therefore, revise your guess.

Delete the number in the B3 cell and enter 54.

B3 | × | ✓ | 54

	A	B
1	Book Problem	
2		
3	Left-hand page number	54
4	Right-hand page number	55
5	Product of page numbers	2970
6		

The computer performs the calculations and different results appear in the B4 and B5 cells.

Step 4: Check your initial guess and revise it if necessary. The product is 2970.

So, the book was open to pages 54 and 55.

Acting Out a Problem

For some problems, you may find it helpful to physically act out the problem situation.

Example

Jon, if you give me one card, I will have as many as you will have.
Matt, if you give me one card, I will have twice as many as you will have.

How many cards does each boy have before the exchange?

Solution

Step 1: Ask yourself these questions:

- What would happen if Jon gave Matt a card? The first condition to be met is that they must have an equal number of cards after the exchange.
- What would happen if Matt gave Jon a card? The second condition to be met is that Jon will have twice as many cards as Matt after the exchange.

Step 2: Make a guess and act out the problem. If your guess does not fit the conditions, revise your guess and repeat the process.

It is helpful to organize your guesses in a table.

Matt's cards	1	2	3	4	5
Jon's cards	3	4	5	6	7

correct guess

So, Matt has 5 cards and Jon has 7 cards.

Note

If Jon gives Matt a card, each boy will have an equal number of cards after the exchange.

Each boy will have 6 cards.

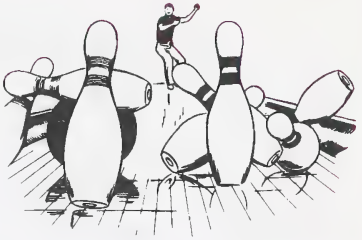
If Matt gives Jon a card, Jon will have twice as many cards as Matt after the exchange.

Jon will have 8 cards and Matt will have 4 cards.

Working Backwards

In mathematics you are usually given information and then asked to find the answer. Sometimes, however, you are given the answer and then you are asked to find a piece of information. In cases like these, you need to work backwards.

Example 1



Ross plays in a bowling league. His average in the league is 198. Today Ross scored 195 and 175 in his first two games. What minimum score must he make in his third game to maintain his average?

Solution

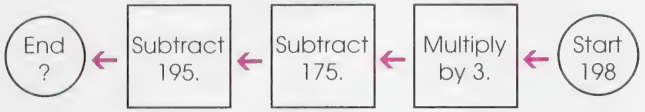
Step 1: Consider how the problem would be done if you worked forwards. This flow chart shows the sequence of the steps.

You start with the score of the third game, add each of the scores of the other two games, divide by 3, and end with the average of 198.

You do not know one of the three scores, so you cannot do the first step, but you can work backwards.

Step 2: Use a reverse flow chart to work backwards.

These operations undo operations in the original flow chart.



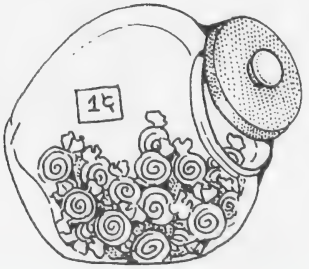
If you start with the average of 198, multiply by 3, subtract 175, and subtract 195, you will end with the minimum score that Ross must get in the third game.

Ross must get at least 224 in the last game.

Note: You can check to see if 224 is the correct third score by using the flow chart in Step 1.

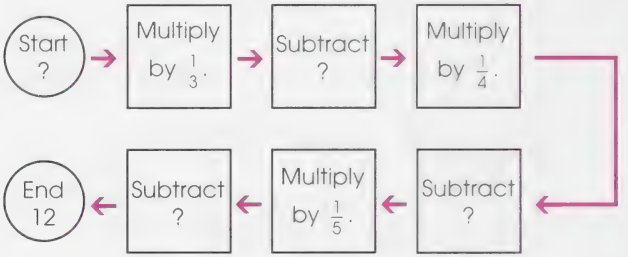
Example 2

Ruth has a jar of candies. She gives Raschid one-third of her candies. She then gives Steven one-fourth of the remaining candies. Finally, she gives one-fifth of the candies that she has left to Rachel. If Ruth has 12 candies left at the end, how many did she have at the beginning?



Solution

Step 1: Use a flow chart to show the order of operations.



This flow chart is **not** helpful because you do not know how much to subtract in the second, fourth, and sixth boxes.

Knowing the following relationships will help you draw a more complete flow chart.

- Giving away one-third of the candies is the same as multiplying the number of candies by two-thirds.

$$1-\frac{1}{3}=\frac{2}{3}$$

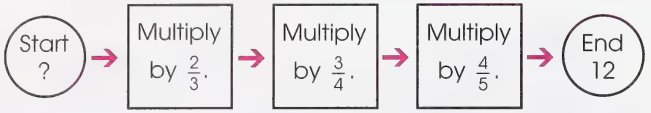
- Giving away one-fourth of the remaining candies is the same as multiplying by three-fourths.

$$1-\frac{1}{4}=\frac{3}{4}$$

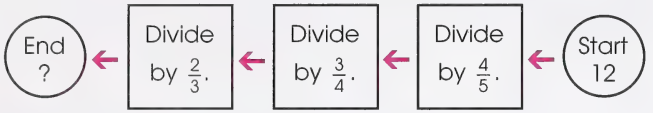
- Giving away one-fifth of the remaining candies is the same as multiplying by four-fifths.

$$1-\frac{1}{5}=\frac{4}{5}$$

The following flow chart is more useful.



Step 2: Use a reverse flow chart to work backwards.



Ruth had 30 candies at the beginning.

Simplifying a Problem

Are you sometimes confused about which operations to perform in a problem because the numbers in the problem are very large or because the problem seems very complicated? There are strategies you can use to simplify a problem.

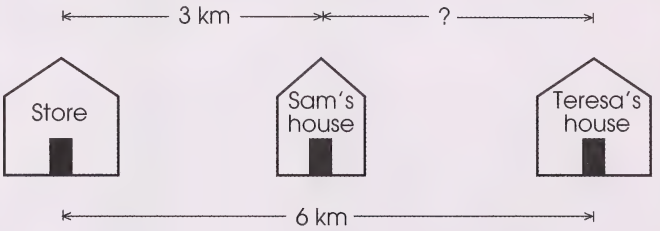
Example

The average distance from the Sun to Mars is 228 000 000 km and the average distance from the Sun to Earth is 150 000 000 km. What is the distance between Earth and Mars?

Solution

Step 1: If you find the large numbers overwhelming, make up a related but simpler problem.

Sam and Teresa both live east of the store. Sam lives 3 km from the store and Teresa lives 6 km from the store. How far is it from Sam's house to Teresa's house?



$6 - 3 = 3$

It is 3 km from Sam's house to Teresa's house.

Step 2: Once you understand the related but simpler problem, you can solve the original problem.

$$228\,000\,000 - 150\,000\,000 = 78\,000\,000$$

The distance from Earth to Mars is 78 000 000 km.

Finding and Applying a Pattern

To solve some problems, it is often helpful to use a related but simpler problem to discover a pattern.

Example

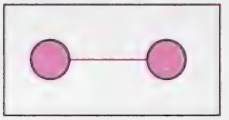
There are 20 students in a room. If every student shakes hands with every other student in the room, how many handshakes are exchanged?

Solution

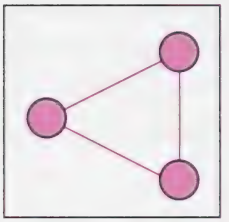
Step 1: Try solving the problem with 1, 2, 3, 4, or 5 students.

- If there is 1 student, no handshakes can be exchanged, as illustrated by the diagram at the right.

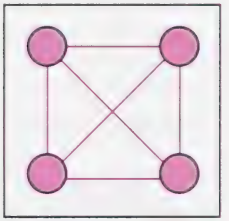
- If there are 2 students, 1 handshake can be exchanged, as illustrated by the diagram at the right.



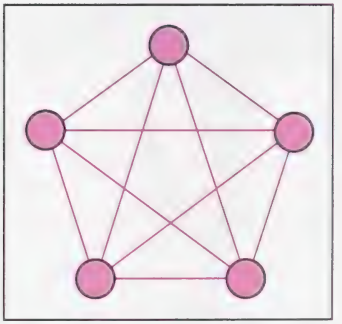
- If there are 3 students, 3 handshakes can be exchanged, as illustrated by the diagram at the right.



- If there are 4 students, 6 handshakes can be exchanged, as illustrated by the diagram at the right.



- If there are 5 students, 10 handshakes can be exchanged, as illustrated by the diagram at the right.

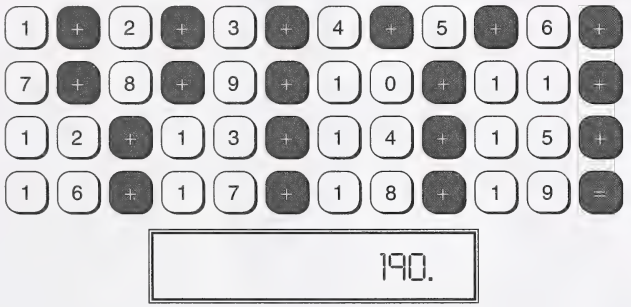


Step 2: Look for a pattern.

People	Handshakes	Pattern
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4

Step 3: Apply the pattern.

A calculator is helpful for applying the pattern.



The total number of handshakes exchanged will be 190.

Using Elimination

Detectives are skilful at eliminating clues and solving mysteries. You can become skilful at using elimination to solve problems too.

Example

Five animals had a race over a short distance. Neither the donkey nor the coyote beat the lion. The coyote beat the rabbit, but not the ostrich. The donkey lagged behind the rabbit. If the ostrich was not the fastest, in what order did the animals finish the race?

Solution

Step 1: Decide what clues are given. The clues indicate the following:

- The donkey and coyote did not finish first. They did not beat the lion.
- The rabbit did not finish first. The coyote beat it.
- The coyote did not finish first. The ostrich was faster.
- The donkey did not finish first. It lagged behind the rabbit.
- The ostrich did not finish first. It was not the fastest.

The clues can be shown on a table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	
Second					
Third					
Fourth					
Fifth					

Step 2: Use elimination and reconsider the clues.

By elimination, it is clear that the lion was first. Now reconsider the clues:

- The rabbit did not finish second. The coyote beat it.
- The coyote did not finish second. The ostrich was faster.
- The donkey did not finish second. It lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗		✗
Third					✗
Fourth					✗
Fifth					✗

Step 3: Use elimination and reconsider the clues.

By elimination, it is clear that the ostrich finished second. Now reconsider the clues:

- The rabbit did not finish third. The coyote beat it.
- The donkey did not finish third. It lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗		✗	✗	✗
Fourth				✗	✗
Fifth				✗	✗

Step 4: Use elimination and reconsider the clues.

By elimination, it is clear that the coyote finished third. Now reconsider the clues:

- The donkey lagged behind the rabbit.

So, the rabbit was fourth and the donkey was fifth.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗	✓	✗	✗	✗
Fourth	✗	✗	✓	✗	✗
Fifth	✓	✗	✗	✗	✗

So, the lion was first, the ostrich was second, the coyote was third, the rabbit was fourth, and the donkey was last.

Using Truth Tables

A strategy that you can use to solve some logic problems is making truth tables. Truth tables are similar to elimination tables.

Example

Detective Beagle is investigating a robbery in the forest. Charlie Chipmunk and Sammy Squirrel each deny being the robber. Harold Hare blames Charlie Chipmunk. Detective Beagle knows two of the animals are lying and one is telling the truth. Who is the robber?

Solution

Step 1: Assume that Charlie Chipmunk was the robber.

- Charlie Chipmunk said he didn't do it. If Charlie Chipmunk was the robber, this is a false statement. Make a truth table and put **F** under Charlie's name.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F		

- Sammy Squirrel denied doing it. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	

- Harold Hare blamed Charlie Chipmunk. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T

Detective Beagle knows two animals are lying. So, Charlie is not the robber.

Step 2: Assume that Sammy Squirrel was the robber.

- Charlie Chipmunk denied doing it. If Sammy Squirrel was the robber, this is a true statement. Put **T** under Charlie's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T		

- Sammy Squirrel denied doing it. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	F	

- Harold Hare blamed Charlie Chipmunk. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	F	F

Detective Beagle knows two of the animals are lying and one is telling the truth.

So, Sammy was the robber.

Using an Equation

A strategy you can use to solve some problems is to write and solve an equation.

Example 1

Twelve less than a number is negative three. Find the number.

Solution

Step 1: Let the unknown be equal to a variable. Let the number be n.

Step 2: Write an equation.

$$n - 12 = -3$$

Twelve less than a number is negative three.

Step 3: Solve the equation.

$$\begin{array}{rcl} n - 12 & = & -3 \\ +12 \quad & & +12 \\ \hline n & = & 9 \end{array}$$

The number is 9.

Example 2

The second of two numbers is seven times the first. The sum of the two numbers is 32. Find the numbers.

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let n be the first number.
Let $7n$ be the second number.

The second of two numbers is seven times the first.

Step 2: Write an equation.

$$n + 7n = 32$$

The sum of the two numbers is 32.

Step 3: Solve the equation.

$$\begin{aligned} n + 7n &= 32 \\ 8n &= 32 \\ \frac{8n}{8} &= \frac{32}{8} \\ n &= 4 \end{aligned}$$

The first number is 4.

Step 4: Find the second number if the first number is 4.

$$\begin{aligned} & 7n \\ &= 7 \times 4 \\ &= 28 \end{aligned}$$

The second number is 28.

So, the numbers are 4 and 28.

Example 3

Yvonne has 20 more nickels than dimes in her piggy bank. If the total value of the nickels and dimes is $8.50, how many coins of each type does she have?

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let the number of dimes be n.

The value of the nickels, the number of dimes, and the value of the dimes must also be represented by algebraic expressions.

A table can be used to organize the information.

Type of Coin	Number of Coins	Value in Cents
Dimes	n	$10n$
Nickels	$n+20$	$5(n+20)$
Total		850

The value of the dimes is 10 times the number of dimes.

The value of the nickels is 5 times the number of nickels.

Yvonne has 20 more nickels than dimes.

This is given.

Step 2: Write an equation.

$$5(n+20)+10n=850$$

Step 3: Solve the equation.

$$\begin{aligned} 5(n+20)+10n &= 850 \\ 5n+100+10n &= 850 \\ 5n+10n+100 &= 850 \\ 15n+100 &= 850 \\ -100 \quad & -100 \\ \hline 15n \quad &= 750 \\ \frac{15n}{15} &= \frac{750}{15} \\ n &= 50 \end{aligned}$$

Yvonne has 50 dimes in her piggy bank.

Step 4: Find the number of nickels if the number of dimes is 50.

$$\begin{aligned} & n+20 \\ &= 50+20 \\ &= 70 \end{aligned}$$

Yvonne has 70 nickels in her piggy bank.

So, Yvonne has 50 dimes and 70 nickels in her piggy bank.

Glossary

Axis of symmetry: a line about which a symmetrical three-dimensional object can be rotated so that its turn image matches its original position more than once in a full turn

Base (of a polyhedron): the face or pair of faces from which the height of the polyhedron is measured

Composite shape: an object composed of a combination of two or more basic shapes

Concave (polyhedron): a polyhedron in which some of the flat surfaces slope inwards

Convex (polyhedron): a polyhedron in which all the flat surfaces slope outwards

Cross section: a two-dimensional figure that is the intersection of a three-dimensional solid and a plane

Edge: a line segment formed by the intersection of two faces

Face: one of the flat surfaces of a polyhedron

Height (of a polyhedron): the vertical distance from a vertex to the base

Net: a two-dimensional pattern that can be folded into a three-dimensional shape

Plane of symmetry: a plane (flat surface) which divides a symmetrical three-dimensional figure into two identical halves

Polyhedron: (plural: **polyhedra**) a three-dimensional shape which has only flat surfaces

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Regular polyhedron: a polyhedron in which all the faces are identical regular polygons and in which the same number of faces and the same number of edges meet at each vertex

Semi-regular polyhedron: a polyhedron in which the faces consist of at least two different types of regular polygons combined in the same way around each vertex; for example, the shapes of a soccer ball

Shell: a three-dimensional model in which the interior is empty

Skeleton: a three-dimensional model showing only the edges and vertices of a three-dimensional shape

Solid: a three-dimensional model in which the interior is filled

Technology: the application of tools, materials, and processes to the solution of problems; more specifically, the devices and systems that are used in processing, transferring, storing, and communicating information through electronic media

Vertex: (plural: **vertices**) a point in a polyhedron where three or more edges meet

Volume: the amount of space an object occupies

Suggested Answers

Section 1: Activity 1

1. **a.** no **b.** yes **c.** yes **d.** no
e. yes **f.** no **g.** yes **h.** yes

2. **a.** N/A **b.** concave **c.** convex **d.** N/A
e. convex **f.** N/A **g.** concave **h.** convex

3. Yes, each of the polyhedra is convex.

4. **a.** Net 1 has 4 vertices and 6 edges.

b. At each vertex, 3 faces and 3 edges meet.

5. **a.** Net 2 has 5 vertices and 9 edges.

b. At 3 vertices, 4 faces and 4 edges meet.
At 2 vertices, 3 faces and 3 edges meet.

6. **a.** Net 3 has 6 vertices and 12 edges.

b. At each vertex, 4 edges and 4 faces meet.

7. **a.** Net 4 has 7 vertices and 15 edges.

b. At 2 vertices, 5 faces and 5 edges meet.
At 5 vertices, 4 faces and 4 edges meet.

8. **a.**

Convex Polyhedra			
Number of Faces	Number of Vertices	Total Number of Faces and Vertices	Number of Edges
4	4	8	6
6	5	11	9
8	6	14	12
10	7	17	15

b. The number of edges is two less than the total number of faces and vertices.

c. $e = f + v - 2$

9. **a.** $e = f + v - 2$
$= 12 + 8 - 2$
$= 20 - 2$
$= 18$

The polyhedron has 18 edges.

b. $e = f + v - 2$
$= 14 + 9 - 2$
$= 23 - 2$
$= 21$

The number of edges is 21.

c.

$$\begin{aligned} e &= f + v - 2 \\ 24 &= 16 + v - 2 \\ 24 &= 14 + v \\ -14 &\quad -14 \\ \hline 10 &= v \end{aligned}$$

The polyhedron has 10 vertices.

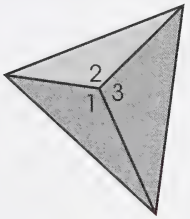
d.

$$\begin{aligned} e &= f + v - 2 \\ 30 &= f + 12 - 2 \\ 30 &= f + 10 \\ -10 &\quad -10 \\ \hline 20 &= f \end{aligned}$$

The number of faces is 20.

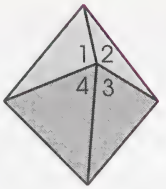
10. The polyhedra made from nets 1, 3, and 8 are regular polyhedra. All the faces are identical regular polygons. The same number of faces and the same number of edges meet at each vertex.

11. **a.** Yes, 3 identical equilateral triangles will form a corner (vertex) of a polyhedron. Because each angle of an equilateral triangle is 60°, the total number of degrees at the vertex is 180°. This is less than 360°.



$60° + 60° + 60° = 180°$

b. Yes, 4 identical equilateral triangles will form a corner (vertex) of a polyhedron. Because each angle of an equilateral triangle is 60°, the total number of degrees at the vertex is 240°. This is less than 360°.



$60° + 60° + 60° + 60° = 240°$

c. Yes, 5 identical equilateral triangles will form a corner (vertex) of a polyhedron. Because each angle of an equilateral triangle is 60°, the total number of degrees at the vertex is 300°. This is less than 360°.

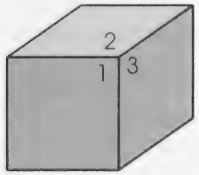
$60° + 60° + 60° + 60° + 60° = 300°$

d. No, 6 identical equilateral triangles will not form a corner (vertex) of a polyhedron. Because each angle of an equilateral triangle is 60°, the total numbers of degrees at the vertex is 360° and the triangles lay flat.

e. No, 7 identical equilateral triangles will not form a corner (vertex) of a polyhedron. The total number of degrees at the vertex is greater than 360°. The triangles overlap.

f. Based on the experiment, 3 regular polyhedra can be formed with equilateral triangles.

12. **a.** Yes, 3 identical squares will form a corner (vertex) of a polyhedron. Because each angle of a square is 90°, the total number of degrees at the vertex is 270°. This is less than 360°.



$90° + 90° + 90° = 270°$

b. No, you cannot form a corner (vertex) of a polyhedron with 4 identical squares. Because each angle of a square is 90°, the total number of degrees at the vertex is 360° and the squares lie flat.

c. No, 5 identical squares will not form a corner (vertex) of a polyhedron. The total number of degrees at the vertex is greater than 360°. The squares overlap.

d. Based on the experiment, 1 regular polyhedron can be formed with square faces.

13. **a.** Yes, 3 identical regular pentagons will form a corner (vertex) of a polyhedron. Because each angle of a regular pentagon is 108°, the total number of degrees at the vertex is 324°. This is less than 360°.

$108° + 108° + 108° = 324°$

b. No, 4 identical regular pentagons will not form a corner (vertex) of a polyhedron. Because each angle of a regular pentagon is 108°, the total number of degrees at the vertex is greater than 360°. The pentagons overlap.

c. Based on the experiment, 1 regular polyhedron can be formed with regular pentagons.

14. **a.** No, you cannot form a corner (vertex) of a polyhedron with 3 identical regular hexagons. Because each angle of a regular hexagon is 120°, the total number of degrees at the vertex is greater than 360°. The hexagons overlap.

b. Based on the experiment, no regular polyhedron can be formed with regular hexagons.

15. **a.** yes **b.** no

16. **a.** There are regular pentagons and regular hexagons on the surface of a soccer ball.

b. There are 3 regular polygons at each vertex—2 hexagons and 1 pentagon. There are 3 edges at each vertex.

c. Yes, a soccer ball has the same height no matter on what face it is seated.

17. **a.** This polyhedron has 2 square faces and 4 rectangular faces.
b. yes

18. **a.** This polyhedron has 1 square face and 4 triangular faces.
b. yes

19. **a.** This polyhedron has 2 hexagonal faces and 6 rectangular faces.
b. yes

20. **a.** This polyhedron has 1 hexagonal face and 6 triangular faces.
b. yes

21. **a.** base **b.** base **c.** "side" face **d.** base
Note: Any face of a rectangular prism can be the base.

22. **a.** base **b.** base **c.** "side" face **d.** base
Note: Any face of a regular triangular pyramid (tetrahedron) can be the base.

23. **a.** The prism in question 21.a. is a triangular prism .
The prism in question 21.b. is a hexagonal prism.
The prism in question 21.c. is a pentagonal prism.
The prism in question 21.d. is a rectangular prism.

b. The pyramid in question 22.a. is an octagonal pyramid.
The pyramid in question 22.b. is a rectangular pyramid.
The pyramid in question 22.c. is a hexagonal pyramid.
The pyramid in question 22.d. is a triangular pyramid.

c. The polyhedron made from Net 9 is a rectangular prism.
The polyhedron made from Net 10 is a square pyramid.
The polyhedron made from Net 11 is a hexagonal prism.
The polyhedron made from Net 12 is a hexagonal pyramid.

24. **a.** pyramid **b.** prism **c.** pyramid **d.** pyramid
e. prism **f.** prism **g.** prism **h.** prism

25. **a.** yes **b.** yes **c.** yes **d.** yes **e.** yes

f. The number of edges is three times the number of sides on the base. The number of vertices is twice the number of sides on the base.

26. **a.** no **b.** no **c.** no **d.** yes

e. The number of edges is twice the number of sides on the base. The number of vertices is one more than the number of sides on the base.

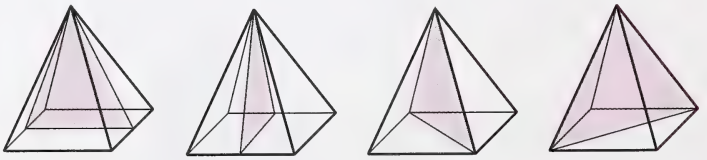
27. **a.** **b.**

c. **d.**

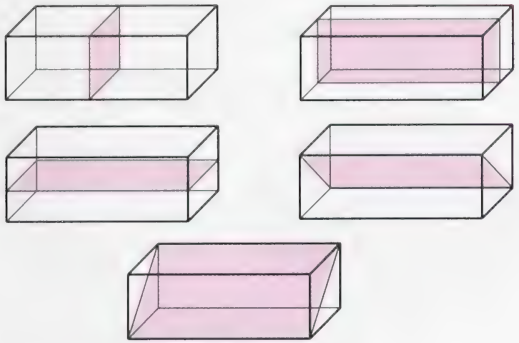
28. **a.** *G* **b.** *H* **c.** *I* **d.** *J* **e.** *K* **f.** *L*

29. **a.** *B* **b.** *C*

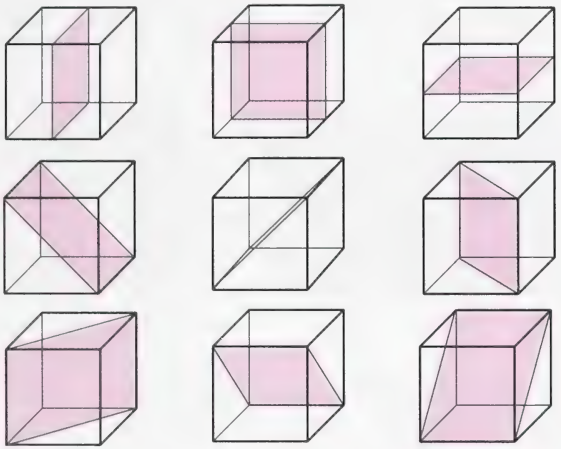
30. **a.** A square pyramid has four planes of symmetry.



b. A rectangular prism has five planes of symmetry.

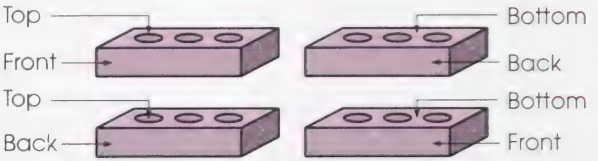


c. A cube has nine planes of symmetry.



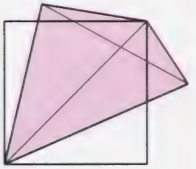
31. **a.** It will be in a matching position four times.
b. It will be in a matching position six times.

32. The brick can be fitted into the space in four ways. The brick can be rotated and flipped.



Now Try This

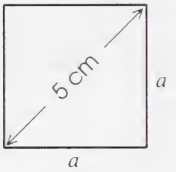
33. If one edge of the tetrahedron is placed along a diagonal of a square hole, the tetrahedron will pass through the hole. Because the edge is 5 cm in length, the smallest possible square hole will have a diagonal of 5 cm.



Use the Pythagorean relation to help you calculate the dimensions of this square hole.

$$a^2 + a^2 = 5^2$$
$$2a^2 = 25$$
$$a^2 = \frac{25}{2}$$
$$a^2 = 12.5$$
$$a \doteq 3.54$$

Round the square root of 12.5 to the nearest hundredth.

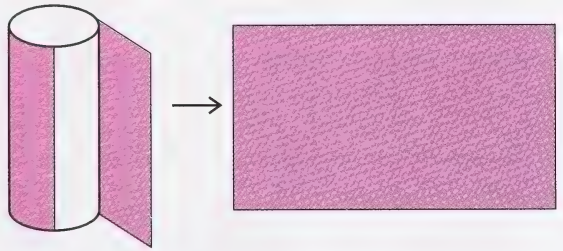


The smallest square will have sides of about 3.54 cm.

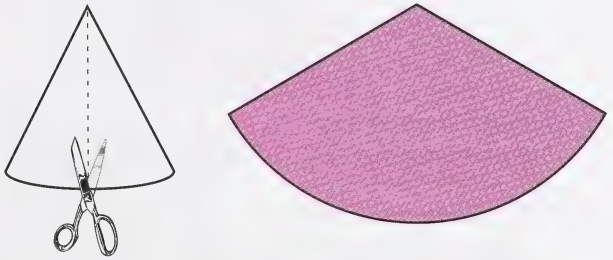
Section 1: Activity 2

1. **a.** cylinder **b.** cone **c.** cylinder
d. sphere **e.** cone **f.** cylinder

2. The label is a rectangle.Peeling the label from an empty can of food illustrates that the curved surface of a cylinder, when flattened, is a rectangle.



3. The hat or a cup is a sector of a circle. Cutting and flattening a conical party hat or cup illustrates the curved surface of a cone is a sector of a circle.



4. The cone is shorter and the circular base has a greater diameter when the central angle is larger. The cone is taller and the circular base has a smaller diameter when the central angle is smaller.

5. **a.** The sides of both a prism and a cylinder are vertical. Each has two bases.

b. The sides of both a pyramid and a cone are slanted. Each has one base.

6. **a.** The length of the rectangle is equal to the circumference of the circular base of the cylinder.

b. The width of the rectangle equals the height of the cylinder.

7. **a.** The cross section is a circle.
b. No, the cross section is always a circle.

8. **a.** **b.** **c.** **d.**

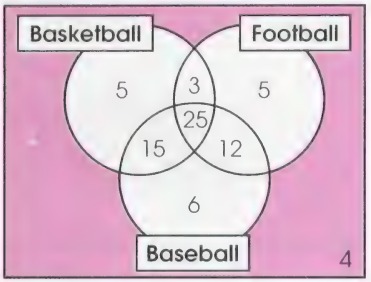
9. **a.** yes **b.** yes **c.** yes **d.** no

10. **a.** no **b.** yes **c.** yes **d.** yes

11. The axis of symmetry passes through the North Pole and the South Pole.

Now Try This

12. You can solve the problem by using a Venn diagram.



$5 + 3 + 5 + 15 + 25 + 12 + 6 = 71$

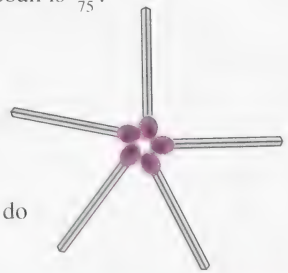
$75 - 71 = 4$

a. Of the students surveyed, 4 students like a sport other than basketball, football, or baseball.

b. The probability that a student selected at random would like basketball, football, or baseball is $\frac{71}{75}$.

13. You can use the guess, check, and revise strategy and objects to solve this problem. Answers will vary.

This is **not** a solution. The matches do not all touch each other.



This is one solution.

Section 1: Follow-up Activities

Extra Help

1. **a.** F **b.** T **c.** F **d.** T **e.** T **f.** F
g. T **h.** F **i.** T **j.** T **k.** T **l.** T

2.

Shape	Number of Faces	Number of Vertices	Number of Edges
Triangular Prism	5	6	9
Triangular Pyramid	4	4	6
Cube	6	8	12
Square Pyramid	5	5	8
Hexagonal Prism	8	12	18
Hexagonal Pyramid	7	7	12

3. **a.** A plane of symmetry is a plane that cuts a polyhedron into two identical pieces so that every point on one side of the plane is balanced by a point on the other side of the plane.

b. An axis of symmetry is a line about which an object can be turned so that in a full turn the turn image will match the original position more than once.

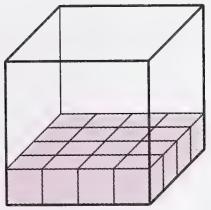
Enrichment

1. a. This dodecahedron is convex.
b. Yes, this dodecahedron is symmetric.

2. a. This stellated dodecahedron is concave.
b. Yes, this stellated dodecahedron is symmetric.

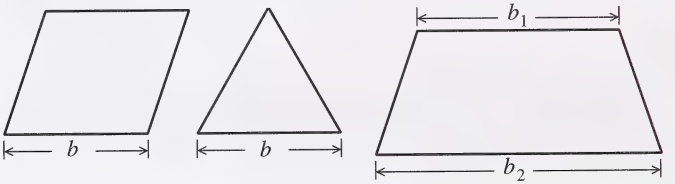
Section 2: Activity 1

1. a. When you use unit cubes to find the volume of a rectangular prism, you count the number of cubes in a single layer and multiply by the number of layers. This is the same as calculating the area of the base $(A = \ell w)$ and multiplying by the height (h).

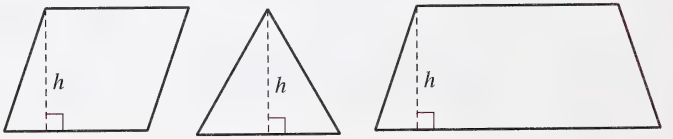


b. When you use unit "cubes" to find the volume of a cylinder, you find the number of "cubes" in a single layer and multiply by the number of layers. This is the same as calculating the area of the base $\left(A = \pi r^2\right)$ and multiplying by the height (h).

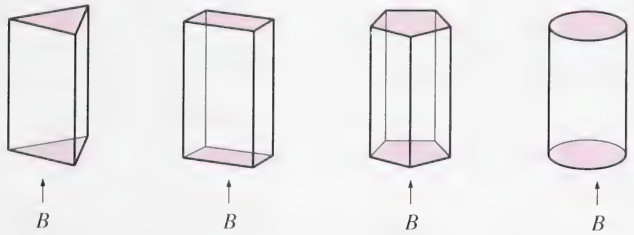
2. a. In these formulas, b represents the length of the bases of the figures.



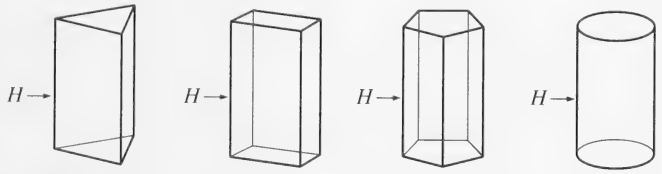
b. In these formulas, h represents the height of the figures.



c. In this formula, B represents the area of the base of the prism or cylinder.



d. In this formula, H represents the height of the prism or cylinder.



3. a. Step 1: Calculate B, the area of the base. **Note:** The base is a triangle.

$$B = \frac{bh}{2}$$
$$= \frac{3(7)}{2}$$
$$= 10.5$$

The area of the base is 10.5 m^2.

Step 2: Calculate V, the volume of the prism.

$$V = BH$$
$$= 10.5(12)$$
$$= 126$$

The volume is 126 m^3.

b. Step 1: Calculate B, the area of the base. **Note:** The base is a circle.

$$B = \pi r^2$$
$$\doteq 3.14\left(8^2\right)$$
$$\doteq 3.14(64) \longleftarrow \text{Use a calculator.}$$
$$\doteq 200.96$$

The area of the base is about 200.96 cm^2.

Step 2: Calculate V, the volume of the cylinder.

$$V = BH$$
$$\doteq 200.96(15) \longleftarrow \text{Use a calculator.}$$
$$\doteq 3014.4$$

The volume is about 3014.4 cm^3.

Note: Your answer may vary slightly if you pressed the π key instead of entering 3.14.

c. **Step 1:** Calculate B, the area of the base. **Note:** The base is a regular hexagon.

$$B = \frac{nsa}{2}$$
$$= \frac{6(0.8)(0.7)}{2} \leftarrow \text{Use a calculator.}$$
$$= 1.68$$

The area of the base is $1.68\ \text{m}^2$.

Step 2: Calculate V, the volume of the prism.

$$V = BH$$
$$= 1.68(3.2) \leftarrow \text{Use a calculator.}$$
$$= 5.376 \leftarrow \text{Round to the nearest tenth.}$$
$$\doteq 5.4$$

The volume is about $5.4\ \text{m}^3$.

d. **Step 1:** Calculate B, the area of the base. **Note:** The base is a rectangle.

$$B = \ell w$$
$$= 20(6)$$
$$= 120$$

The area of the base is $120\ \text{mm}^2$.

Step 2: Calculate V, the volume of the prism.

$$V = BH$$
$$= 120(14) \leftarrow \text{Use a calculator.}$$
$$= 1680$$

The volume is $1680\ \text{mm}^3$.

e. **Step 1:** Calculate B, the area of the base. **Note:** The base is a triangle.

$$B = \frac{bh}{2}$$
$$= \frac{12(8)}{2}$$
$$= \frac{96}{2}$$
$$= 48$$

The area of the base is $48\ \text{cm}^2$.

Step 2: Calculate V, the volume of the prism.

$$V = BH$$
$$= 48(22) \leftarrow \text{Use a calculator.}$$
$$= 1056$$

The volume is $1056\ \text{cm}^3$.

f. **Step 1:** Calculate B, the area of the base. **Note:** The base is a circle.

$$\begin{aligned} B &= \pi r^2 \\ &\doteq 3.14\left(6^2\right) \\ &\doteq 3.14(36) \quad \leftarrow \text{Use a calculator.} \\ &\doteq 113.04 \end{aligned}$$

$d = 12$ m
$r = 6$ m

The area of the base is about 113.04 m^2.

Step 2: Calculate V, the volume of the cylinder.

$$\begin{aligned} V &= BH \\ &\doteq 113.04(6) \quad \leftarrow \text{Use a calculator.} \\ &\doteq 678.24 \quad \leftarrow \text{Round to the nearest tenth.} \\ &\doteq 678.2 \end{aligned}$$

The volume is about 678.2 m^3.

4. **Step 1:** Calculate B, the area of the base. **Note:** The base is a regular decagon.

$$\begin{aligned} B &= \frac{nsa}{2} \\ &= \frac{10(5.0)(7.4)}{2} \\ &= 185 \quad \leftarrow \text{Use a calculator.} \end{aligned}$$

The area of the base is 185 cm^2.

Step 2: Calculate V, the volume of the prism.

$$\begin{aligned} V &= BH \\ &= 185(5.8) \quad \leftarrow \text{Use a calculator.} \\ &= 1073 \end{aligned}$$

The volume is 1073 cm^3.

5. **Step 1:** Calculate B, the area of the base. **Note:** The base is a hendecagon.

$$\begin{aligned} B &= \frac{nsa}{2} \\ &= \frac{11(8.0)(13.3)}{2} \quad \leftarrow \text{Use a calculator.} \\ &= 585.2 \end{aligned}$$

The area of the base is 585.2 mm^2.

Step 2: Calculate V, the volume of the prism.

$$\begin{aligned} V &= BH \\ &= 585.2(2.0) \\ &= 1170.4 \end{aligned}$$

The volume is 1170.4 mm^3.

6. **Step 1:** Calculate B, the area of the base. **Note:** The base is a circle.

$$\begin{aligned} B &= \pi r^2 \\ &\doteq 3.14\left(5^2\right) \\ &\doteq 3.14(25) \quad \longleftarrow \text{Use a calculator.} \\ &\doteq 78.5 \end{aligned}$$

The area of the base is about 78.5 cm^2.

Step 2: Calculate V, the volume of the cylinder.

$$\begin{aligned} V &= BH \\ &\doteq 78.5(12) \quad \longleftarrow \text{Use a calculator.} \\ &\doteq 942 \end{aligned}$$

The volume is about 942 cm^3.

Note: Your answer may vary slightly if you pressed the π key instead of entering 3.14.

7. **Step 1:** Find the volume of the small cylinder (the cap of the bottle). **Hint:** If the diameter is 2 cm, the radius is 1 cm.

$$\begin{aligned} B &= \pi r^2 \\ &\doteq 3.14\left(1^2\right) \\ &\doteq 3.14(1) \\ &\doteq 3.14 \end{aligned} \qquad \begin{aligned} V &= BH \\ &\doteq 3.14(2) \\ &\doteq 6.28 \\ &\doteq 6.3 \end{aligned}$$

The volume of the small cylinder is about 6.3 cm^3.

Step 2: Find the volume of the large cylinder. **Hint:** If the diameter is 4 cm, the radius is 2 cm.

$$\begin{aligned} B &= \pi r^2 \\ &\doteq 3.14\left(2^2\right) \\ &\doteq 3.14(4) \\ &\doteq 12.56 \end{aligned} \qquad \begin{aligned} V &= BH \\ &\doteq 12.56(6) \\ &\doteq 75.36 \\ &\doteq 75.4 \end{aligned}$$

The volume of the large cylinder is about 75.4 cm^3.

Step 3: Find the volume of the composite shape.

$$6.3 + 75.4 = 81.7$$

The volume of the composite shape is about 81.7 cm^3.

Note: Your answer may vary slightly if you pressed the π key instead of entering 3.14.

8. **Step 1:** Find the volume of the top. **Hint:** The top is one-half of a cylinder. Therefore, find the volume of a cylinder with a radius of 15 cm and a height of 30 cm; then divide by 2.

$$\begin{aligned} B &= \pi r^2 \\ &\doteq 3.14\left(15^2\right) \\ &\doteq 3.14(225) \\ &\doteq 706.5 \end{aligned} \qquad \begin{aligned} V &= BH \\ &\doteq 706.5(30) \\ &\doteq 21\,195 \end{aligned}$$

$21\,195 \div 2 = 10\,597.5$

The volume of the top is about 10 597.5 cm^3.

Step 2: Find the volume of the bottom. It is a rectangular prism.

$$B = \ell w \qquad V = BH$$
$$= 30(15) \qquad = 450(15)$$
$$= 450 \qquad = 6750$$

The volume of the bottom is 6750 cm^3.

Step 3: Find the volume of the composite shape.

$$10\,597.5 + 6750 = 17\,347.5$$

The volume of the composite shape is about 17 347.5 cm^3.

9. a. Step 1: Find the volume of the hexagonal prism.

$$B = \frac{nsa}{2} \qquad V = BH$$
$$= \frac{6(7)(7)}{2} \qquad = 147(6)$$
$$= \frac{294}{2} \qquad = 882$$
$$= 147$$

The volume of the hexagonal prism is 882 mm^3.

Step 2: Find the volume of the cylindrical hole. **Hint:** If the diameter is 5 mm, the radius is 2.5 mm.

$$B = \pi r^2 \qquad V = BH$$
$$\doteq 3.14(2.5)^2 \qquad \doteq 19.625(6)$$
$$\doteq 3.14(6.25) \qquad \doteq 117.75$$
$$\doteq 19.625 \qquad \doteq 117.8$$

The volume of the cylindrical hole is about 1117.8 mm^3.

Step 3: Find the volume of the composite shape.

$$882 - 117.8 = 764.2$$

The volume of the composite shape is about 764.2 mm^3.

b. Step 1: Find the volume of the cylindrical portion. **Hint:** If the diameter is 1.5 cm, the radius is 0.75 cm.

$$B = \pi r^2 \qquad V = BH$$
$$\doteq 3.14\left(0.75^2\right) \qquad \doteq 1.76625(5.0)$$
$$\doteq 3.14(0.5625) \qquad \doteq 8.831\,25$$
$$\doteq 1.766\,25 \qquad \doteq 8.8$$

The volume of the cylindrical portion is about 8.8 cm^3.

Step 2: Find the volume of the hexagonal prism.

$$B = \frac{nsa}{2} \qquad V = BH$$
$$= \frac{6(0.8)(0.8)}{2} \qquad = 1.92(1.0)$$
$$= \frac{3.84}{2} \qquad = 1.92$$
$$= 1.92 \qquad \doteq 1.9$$

The volume of the hexagonal portion is about $1.9\ \text{cm}^3$.

Step 3: Find the volume of the composite shape.

$$8.8 + 1.9 = 10.7$$

The volume of the composite shape is about $10.7\ \text{cm}^3$.

10. a. The minimum dimensions of the box required to package the teddy bear are $16\ \text{cm} \times 12\ \text{cm} \times 35\ \text{cm}$.

The expression $16\ \text{cm} \times 12\ \text{cm} \times 35\ \text{cm}$ is read as, "16 cm by 12 cm by 35 cm."

b. $B = \ell w$ $\qquad V = BH$
$= 16(12)$ $\qquad = 192(35)$
$= 192$ $\qquad = 6720$

The minimum volume of the box is $6720\ \text{cm}^3$.

11. a. The minimum dimensions of the box required to package the chair are $1.0\ \text{m} \times 0.6\ \text{m} \times 0.7\ \text{m}$.

b. $B = \ell w$ $\qquad V = BH$
$= 1.0 \times 0.6$ $\qquad = 0.6(0.7)$
$= 0.6$ $\qquad = 0.42$

The minimum volume of the box is $0.42\ \text{m}^3$.

12. Step 1: Calculate the volume of the carton.

$B = \ell w$ $\qquad V = BH$
$= 36(18)$ $\qquad = 648(11)$
$= 648$ $\qquad = 7128$

The volume of the box is $7128\ \text{cm}^3$.

Step 2: Calculate the volume of each can. **Hint:** If the diameter is 6 cm, the radius is 3 cm.

$B = \pi r^2$ $\qquad V = BH$
$\doteq 3.14\left(3^2\right)$ $\qquad \doteq 28.26(11)$
$\doteq 3.14(9)$ $\qquad \doteq 310.86$
$\doteq 28.26$ $\qquad \doteq 310.9$

The volume of each can is about $310.9\ \text{cm}^3$.

Step 3: Calculate the volume of 18 cans.

$$18 \times 310.9 = 5596.2$$

The volume of 18 cans is about 5596.2 cm^3.

Step 4: Calculate the volume of empty space.

$$7128 - 5596.2 = 1531.8$$

The volume of the empty space is about 1531.8 cm^3.

13. a.

$B = \ell w$ $\quad$ $V = BH$

$= 26(16)$ $\quad$ $= 416(13)$

$= 416$ $\quad$ $= 5408$

The volume of the radio is 5408 cm^3.

b.

$B = \ell w$ $\quad$ $V = BH$

$= 30(20)$ $\quad$ $= 600(16)$

$= 600$ $\quad$ $= 9600$

The volume of the box is 9600 cm^3.

c. $9600 - 5408 = 4192$

The volume of the styrofoam is about 4192 cm^3.

14. a. Find the area of the base of the box and then calculate the volume.

$B = \ell w$ $\quad$ $V = BH$

$= 100(50)$ $\quad$ $= 5000(50)$

$= 5000$ $\quad$ $= 250\,000$

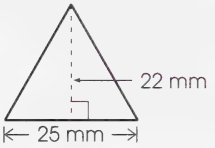
The volume of the box is 250 000 mm^3.

b. **Step 1:** Find the area of the base of the triangular prism.

$$A = \frac{bh}{2}$$

$$= \frac{25(22)}{2}$$

$$= 275$$



The area of the base of the triangular prism is 275 mm^2.

Step 2: Find the volume of the triangular prism.

$$V = BH$$

$$= 275(75)$$

$$= 20\,625$$

The volume of the triangular prism is about 20 625 mm^3.

c. Find the volume of the styrofoam chips. The volume of the styrofoam chips will be equal to the volume of the box minus the volume of the prism.

$$250\,000 - 20\,344 = 229\,656$$

The volume of the styrofoam chips is about 229 656 mm^3.

15. a. The triangular prism is the most unusual. (Most containers are cylinders or rectangular prisms.)

b. It is probably easiest to grip the cylindrical container.

c. **Step 1:** Calculate the volume of the triangular prism.

$$B = \frac{bh}{2} \qquad V = BH$$
$$= \frac{8(8)}{2} \qquad = 32(18)$$
$$= \frac{64}{2} \qquad = 576$$
$$= 32$$

The volume of the triangular prism is 576 cm^3.

Step 2: Calculate the volume of the cylinder.

$$B = \pi r^2 \qquad V = BH$$
$$\doteq 3.14\left(4^2\right) \qquad \doteq 50.24(12)$$
$$\doteq 3.14(16) \qquad \doteq 602.88$$
$$\doteq 50.24 \qquad \doteq 602.9$$

The volume of the cylinder is about 602.9 cm^3.

Step 3: Calculate the volume of the rectangular prism.

$$B = \ell w \qquad V = BH$$
$$= 8(8) \qquad = 64(12)$$
$$= 64 \qquad = 768$$

The volume of the rectangular prism is 768 cm^3.

Step 4: Compare the volumes and decide which is largest.

$$768 > 602.9 > 576$$

The rectangular prism has the greatest volume.

16. a.
$$B = \pi r^2 \qquad V = BH$$
$$\doteq 3.14\left(5^2\right) \qquad \doteq 78.5(8)$$
$$\doteq 3.14(25) \qquad \doteq 628$$
$$\doteq 78.5$$

The jar has a volume of about 628 cm^3.

b. $B = \pi r^2$

$\doteq 3.14\left(4^2\right)$

$\doteq 3.14(16)$

$\doteq 50.24$

$V = BH$

$\doteq 50.24(7)$

$\doteq 351.68$

$\doteq 351.7$

The volume of the space for the facial cream is about $351.7\ \text{cm}^3$.

c. Answers will vary. The walls are probably thick to make it appear that there is more facial cream. A container with thicker walls may also be less fragile.

Now Try This

17. You can use logical reasoning to solve the problem.

Step 1: Calculate the number of wagons left after 12 wagons broke down.

$$96 - 12 = 84$$

After the breakdown, 84 wagons were left.

Step 2: Calculate the number of people that were transferred from the 12 broken-down wagons. **Hint:** After the 12 wagons broke down, each of the remaining 84 wagons had one more person.

There were 84 people transferred from the 12 broken-down wagons.

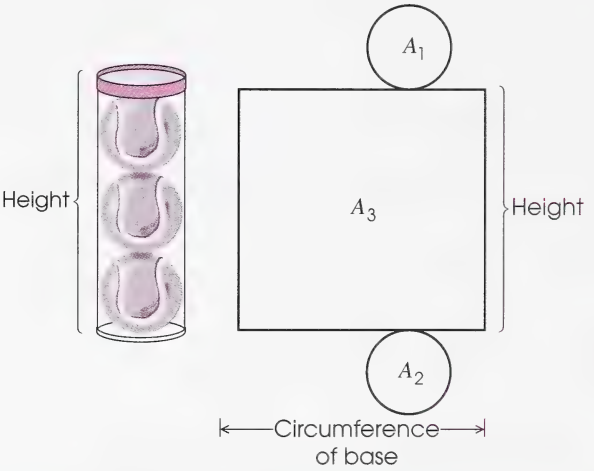
Step 3: Calculate the number of people that were originally in each wagon. **Hint:** Each wagon had the same number of people and 84 people were transferred from 12 wagons.

$$84 \div 12 = 7$$

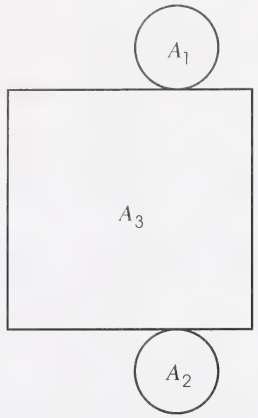
Each wagon originally had 7 people.

Section 2: Activity 2

1. The surface area of the can was calculated by finding the area of the three different surfaces making up the can and then adding these areas. **Note:** The top and bottom of the can are identical circles. The curved surface of the can, when flattened, is a rectangle. The length of the rectangle is equal to the circumference of the circular base. The width of the rectangle is equal to the height of the can.



2. Use this reasoning. The surface area of each cylinder is equal to the sum of the following areas. **Note:** $A_1 = A_2$.



a. **Step 1:** Find the area of A_1 and A_2. **Note:** Each circular base has a diameter of 12 cm and a radius of 6 cm.

$$\begin{aligned} A_1 &= \pi r^2 \\ &\doteq 3.14\left(6^2\right) \\ &\doteq 3.14(36) \\ &\doteq 113.04 \end{aligned}$$

$$\therefore\ A_2 \doteq 113.04$$

and

$$A_1 + A_2 \doteq 226.08$$

The area of A_1 and A_2 is about 226.08 cm^2.

Step 2: Find the area of A_3. **Note:** The curved surface is a rectangle. The length of the rectangle is πd or about 37.68 cm. The width of the rectangle is 24 cm.

$$\begin{aligned} A_3 &= \ell w \\ &\doteq 37.68(24) \\ &\doteq 904.32 \end{aligned}$$

The area of A_3 is about 904.32 cm^2.

Step 3: Find the surface area of the cylinder.

$$\begin{aligned} \left(A_1 + A_2\right) + A_3 &\doteq 226.08 + 904.32 \\ &\doteq 1130.4 \end{aligned}$$

The surface area of the can is about 1130.4 cm^2.

b. **Step 1:** Find the area of A_1 and A_2. **Note:** Each circular base has a diameter of 6 m and a radius of 3 m.

$$\begin{aligned} A_1 &= \pi r^2 \\ &\doteq 3.14\left(3^2\right) \\ &\doteq 3.14(9) \\ &\doteq 28.26 \end{aligned}$$

$$\therefore\ A_2 \doteq 28.26$$

and

$$A_1 + A_2 \doteq 56.52$$

The area of A_1 and A_2 is about 56.52 m^2.

Step 2: Find the area of A_3. **Note:** The curved surface is a rectangle. The length of the rectangle is πd or about 18.84 m. The width of the rectangle is 10 m.

$$\begin{aligned} A_3 &= \ell w \\ &\doteq 18.84(10) \\ &\doteq 188.4 \end{aligned}$$

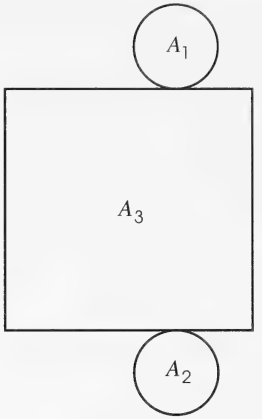
The area of A_3 is about 188.4 m^2.

Step 3: Find the surface area of the cylinder.

$$\begin{aligned} (A_1 + A_2) + A_3 &\doteq 56.52 + 188.4 \\ &\doteq 244.92 \\ &\doteq 244.9 \end{aligned}$$

The surface area of the cylinder is about 244.9 m^2.

3. Use this reasoning. The surface area is the sum of the following areas.



Step 1: Find the area of A_1 and A_2. **Note:** $A_1 = A_2$.

$$\begin{aligned} A_1 &= \pi r^2 \\ &\doteq 3.14\left(3^2\right) \\ &\doteq 3.14(9) \\ &\doteq 28.26 \end{aligned}$$

$\therefore\ A_2 \doteq 28.26$

and

$A_1 + A_2 \doteq 56.52$

The area of A_1 and A_2 is about 56.52 cm^2.

Step 2: Find the surface area of A_3. **Note:** The curved surface is a rectangle. The length of the rectangle is equal to πd or about 18.84 cm. The width of the rectangle is 12 cm.

$$\begin{aligned} A_3 &= \ell w \\ &\doteq 18.84(12) \\ &\doteq 226.08 \end{aligned}$$

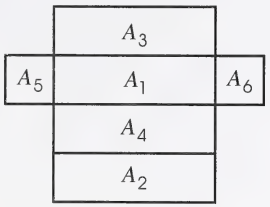
The area of A_3 is about 226.08 cm^2.

Step 3: Find the surface area of the can.

$$\begin{aligned} (A_1 + A_2) + A_3 &\doteq 56.52 + 226.08 \\ &\doteq 282.6 \end{aligned}$$

The surface area of the can is about 282.6 cm^2.

4. a. Use this reasoning. The surface area of the box is the sum of these 6 areas. **Note:** $A_1 = A_2$, $A_3 = A_4$, $A_5 = A_6$.



Step 1: Find the area of A_1 and A_2. **Note:** Each rectangle is 17.8 cm × 12.4 cm.

$$\begin{aligned} A_1 &= \ell w \\ &= 17.8(12.4) \\ &= 220.72 \end{aligned} \qquad \begin{aligned} &\therefore\ A_2 = 220.72 \\ &\text{and} \\ &A_1 + A_2 = 441.44 \end{aligned}$$

The area of A_1 and A_2 together is 441.44 cm^2.

Step 2: Find the area of A_3 and A_4. **Note:** Each rectangle is 17.8 cm × 6.0 cm.

$$\begin{aligned} A_3 &= \ell w \\ &= 17.8(6.0) \\ &= 106.8 \end{aligned} \qquad \begin{aligned} &\therefore\ A_4 = 106.8 \\ &\text{and} \\ &A_3 + A_4 = 213.6 \end{aligned}$$

The area of A_3 and A_4 together is 213.6 cm^2.

Step 3: Find the area of A_5 and A_6. **Note:** Each rectangle is 12.4 cm × 6.0 cm.

$$\begin{aligned} A_5 &= \ell w \\ &= 12.4(6.0) \\ &= 74.4 \end{aligned} \qquad \begin{aligned} &\therefore\ A_6 = 74.4 \\ &\text{and} \\ &A_5 + A_6 = 148.8 \end{aligned}$$

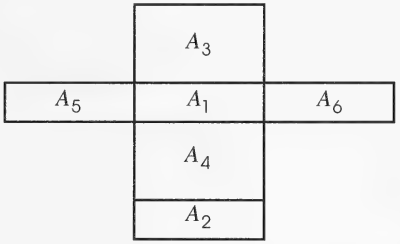
The area of A_5 and A_6 together is 148.8 cm^2.

Step 4: Find the surface area of the box.

$$\begin{aligned}&(A_1 + A_2) + (A_3 + A_4) + (A_5 + A_6)\\ &= 441.44 + 213.6 + 148.8\\ &= 803.84\end{aligned}$$

The surface area is 803.84 cm^2.

b. The surface area is the sum of these 6 areas. **Note:** $A_1 = A_2$, $A_3 = A_4$, $A_5 = A_6$.



Step 1: Find the area of A_1 and A_2. **Note:** Each rectangle is 8.0 cm × 4.5 cm.

$$\begin{aligned}A_1 &= \ell w & \therefore\ A_2 &= 36\\ &= 8.0(4.5) & \text{and}\\ &= 36 & A_1 + A_2 &= 72\end{aligned}$$

The area of A_1 and A_2 together is 72 cm^2.

Step 2: Find the area of A_3 and A_4. **Note:** Each rectangle is 13.0 cm × 8.0 cm.

$$\begin{aligned}A_3 &= \ell w & \therefore\ A_4 &= 104\\ &= 13.0(8.0) & \text{and}\\ &= 104 & A_3 + A_4 &= 208\end{aligned}$$

The area of A_3 and A_4 together is 208 cm^2.

Step 3: Find the area of A_5 and A_6. **Note:** Each rectangle is 13.0 cm × 4.5 cm.

$$\begin{aligned}A_5 &= \ell w & \therefore\ A_6 &= 58.5\\ &= 13.0(4.5) & \text{and}\\ &= 58.5 & A_5 + A_6 &= 117\end{aligned}$$

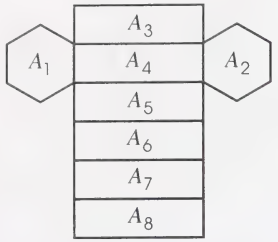
The area of A_5 and A_6 together is 117 cm^2.

Step 4: Find the surface area of the box.

$$\begin{aligned}&(A_1 + A_2) + (A_3 + A_4) + (A_5 + A_6)\\ &= 72 + 208 + 117\\ &= 397\end{aligned}$$

The surface area is 397 cm^2.

5. The surface area is the sum of these 8 areas. **Note:** $A_1 = A_2$, $A_3 = A_4 = A_5 = A_6 = A_7 = A_8$.



Step 1: Find the area of A_1 and A_2. **Note:** Each hexagon has 6 sides of 7.5 cm. The apothem is 8 cm.

$$A_1 = \frac{nsa}{2}$$
$$= \frac{6(7.5)(8.0)}{2}$$
$$= \frac{360}{2}$$
$$= 180$$

$\therefore\ A_2 = 180$

and

$A_1 + A_2 = 360$

The area of A_1 and A_2 together is 360 cm^2.

Step 2: Find the area of A_3, A_4, A_5, A_6, A_7, and A_8. **Note:** Each rectangle is 7.5 cm × 5.8 cm.

$$A_3 = \ell w$$
$$= 7.5(5.8)$$
$$= 43.5$$

$\therefore\ A_3 = 43.5$

$A_4 = 43.5$

$A_5 = 43.5$

$A_6 = 43.5$

$A_7 = 43.5$

$A_8 = 43.5$

and

$A_3 + A_4 + A_5 + A_6 + A_7 + A_8 = 261$

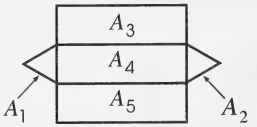
The area of A_3, A_4, A_5, A_6, A_7, and A_8 is 261 cm^2.

Step 3: Find the surface area of the box.

$$(A_1 + A_2) + (A_3 + A_4 + A_5 + A_6 + A_7 + A_8)$$
$$= 360 + 261$$
$$= 621$$

The surface area is 621 cm^2.

6. The surface area is the sum of these 5 areas. **Note:** $A_1 = A_2$, $A_3 = A_4 = A_5$.



Step 1: Find the area of A_1 and A_2. **Note:** Each triangle has a base of 3.0 cm and a height of 2.5 cm.

$$A_1 = \frac{bh}{2}$$
$$= \frac{3.0(2.5)}{2}$$
$$= \frac{7.5}{2}$$
$$= 3.75$$

$\therefore A_2 = 3.75$

and

$A_1 + A_2 = 7.5$

The area of A_1 and A_2 together is 7.5 cm^2.

Step 2: Find the area of A_3, A_4, and A_5. **Note:** Each rectangle is 3.0 cm × 16.7 cm.

$$A_3 = \ell w$$
$$= 16.7(3.0)$$
$$= 50.1$$

$\therefore A_4 = 50.1$

$A_5 = 50.1$

and

$A_3 + A_4 + A_5 = 150.3$

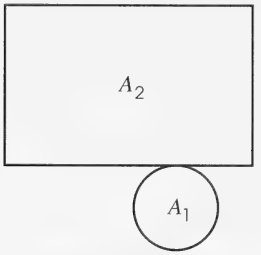
The area of A_3, A_4, and A_5 together is 150.3 cm^2.

Step 3: Find the surface area of the box.

$$(A_1 + A_2) + (A_3 + A_4 + A_5) = 7.5 + 150.3$$
$$= 157.8$$

The surface area is 157.8 cm^2.

7. The surface area of the liner is the sum of these 2 areas.



Step 1: Calculate the area of A_1. **Note:** The circle has a radius of 2 m.

$$A_1 = \pi r^2$$
$$\doteq 3.14(2^2)$$
$$\doteq 3.14(4)$$
$$\doteq 12.56$$

The area of A_1 is about 12.56 m^2.

Step 2: Calculate the area of A_2. **Note:** The rectangle has a length of πd or about 12.56 m. The width is 2 m.

$$A_2 = \ell w$$
$$\doteq 12.56(2)$$
$$\doteq 25.12$$

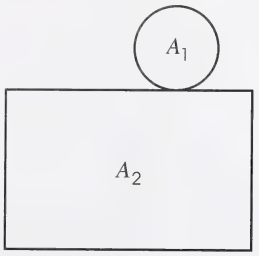
The area of A_2 is 25.12 m^2.

Step 3: Calculate the surface area of the liner.

$$A_1 + A_2 \doteq 12.56 + 25.12$$
$$\doteq 37.68$$

The surface area of the liner is about 37.68 m^2.

8. a. The area of the surfaces of the cake that are iced is the sum of these areas.



Step 1: Calculate the area of A_1. **Note:** The circle has a radius of 10 cm.

$$A_1 = \pi r^2$$
$$\doteq 3.14\left(10^2\right)$$
$$\doteq 3.14(100)$$
$$\doteq 314$$

The area of A_1 is about 314 cm^2.

Step 2: Calculate the area of A_2. **Note:** The rectangle has a length of πd or about 62.8 cm. The width is 13 cm.

$$A_2 = \ell w$$
$$\doteq 62.8(13)$$
$$\doteq 816.4$$

The area of A_2 is about 816.4 cm.

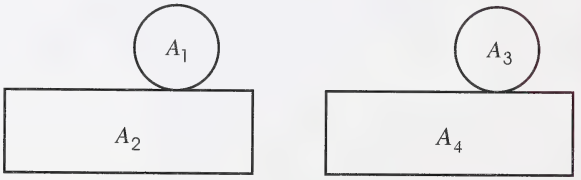
Step 3: Find the area of the surfaces of the cake that are iced.

$$A_1 + A_2 \doteq 314 + 816.4$$
$$\doteq 1130.4$$

The area of the surfaces of the cake that are iced is about 1130.4 cm^2.

b. Use this reasoning. Because the cake has been cut in half, each rectangle has a height of 6.5 cm.

The area of the surfaces of the cake that are iced is the sum of these 4 areas. **Note:** $A_1 = A_3$; $A_2 = A_4$.



Step 1: Calculate the area of A_1 and A_3. **Note:** Each circle has a radius of 10 cm.

$$\begin{aligned} A_1 &= \pi r^2 \\ &\doteq 3.14\left(10^2\right) \\ &\doteq 3.14(100) \\ &\doteq 314 \end{aligned}$$

$$\therefore\ A_3 \doteq 314$$

and

$$A_1 + A_3 \doteq 628$$

The area of A_1 and A_3 is about 628 cm^2.

Step 2: Calculate the area of A_2 and A_4. **Note:** Each rectangle has a length of πd or about 62.8 cm. Each rectangle has a width of 6.5 cm.

$$\begin{aligned} A_2 &= \ell w \\ &\doteq 62.8(6.5) \\ &\doteq 408.2 \end{aligned}$$

$$\therefore\ A_4 \doteq 408.2$$

and

$$A_2 + A_4 \doteq 816.4$$

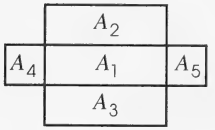
The area of A_2 and A_4 is about 816.4 cm^2.

Step 3: Find the area of the iced surfaces.

$$\begin{aligned} \left(A_1 + A_3\right) + \left(A_2 + A_4\right) &\doteq 628 + 816.4 \\ &\doteq 1444.4 \end{aligned}$$

The area of the iced surfaces is about 1444.4 cm^2.

9. The surface area is the sum of these 5 areas. **Note:** $A_2 = A_3$, $A_4 = A_5$.



Step 1: Find the area of the ceiling (A_1). **Note:** The rectangle is 8 m × 4 m.

$$\begin{aligned} A_1 &= \ell w \\ &= 8(4) \\ &= 32 \end{aligned}$$

The area of the ceiling $\left(A_1\right)$ is 32 m^2.

Step 2: Calculate the area of the long walls (A_2 and A_3). **Note:** Each rectangle is 8 m × 3 m.

$$\begin{aligned} A_2 &= \ell w \\ &= 8(3) \\ &= 24 \end{aligned}$$

$$\therefore\ A_3 = 24$$

and

$$A_2 + A_3 = 48$$

The area of A_2 and A_3 together is 48 m^2.

Step 3: Calculate the area of the short walls (A_4 and A_5). **Note:** Each rectangle is 4 m × 3 m.

$$A_4 = \ell w \qquad \therefore\ A_5 = 12$$
$$= 4(3) \qquad \text{and}$$
$$= 12 \qquad A_4 + A_5 = 24$$

Together the area of A_4 and A_5 is 24 m^2.

Step 4: Calculate the area of the painted surfaces.

$$A_1 + (A_2 + A_3) + (A_4 + A_5) = 32 + 48 + 24$$
$$= 104$$

The area of the painted surfaces is 104 m^2.

Step 5: Calculate the amount of paint required. **Note:** If the amount of paint is a fraction of a litre, you must round the answer **up** to the nearest whole number.

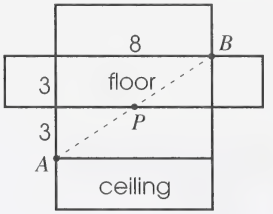
$$104 \div 12 \doteq 9$$

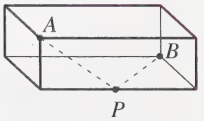
To paint the ceiling and walls 9 L of paint are required.

Now Try This

10. You may use a net and the diagram to help you solve the problem.

Step 1: To find the shortest path, you can draw a sketch of the net.

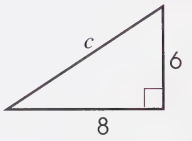




The shortest path is from A to P to B. That is, the spider crawls down the wall to the floor at point P and then across the floor to vertex B.

Step 2: Use the Pythagorean relation to calculate the length of the path. **Note:** The path, as shown on the net (see Step 1), is the hypotenuse of a triangle that has legs of 6 m and 8 m.

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = c^2$$
$$36 + 64 = c^2$$
$$100 = c^2$$
$$10 = c$$



The shortest distance the spider would travel is 10 m.

Section 2: Follow-up Activities

Extra Help

1. a. Step 1: Calculate the area of the base.

$$\begin{aligned} B &= \pi r^2 \\ &\doteq 3.14\left(5^2\right) \\ &\doteq 3.14(25) \\ &\doteq 78.5 \end{aligned}$$

The area of the base is about $78.5\ \text{cm}^2$.

Step 2: Calculate the circumference of the base.

$$\begin{aligned} C &= \pi d \\ &\doteq 3.14(10) \\ &\doteq 31.4 \end{aligned}$$

The circumference of the base is about 31.4 cm.

Step 3: Calculate the surface area.

$$\begin{aligned} SA &= 2B + CH \\ &\doteq 2(78.5) + 31.4(8) \\ &\doteq 157 + 251.2 \\ &\doteq 408.2 \\ &\doteq 408 \end{aligned}$$

The surface area is about $408\ \text{cm}^2$.

b. Step 1: Calculate the area of the base.

$$\begin{aligned} B &= \pi r^2 \\ &\doteq 3.14\left(4^2\right) \\ &\doteq 3.14(16) \\ &\doteq 50.24 \end{aligned}$$

The area of the base is about $50.24\ \text{m}^2$.

Step 2: Calculate the circumference of the base.

$$\begin{aligned} C &= \pi d \\ &\doteq 3.14(8) \\ &\doteq 25.12 \end{aligned}$$

The circumference of the base is about 25.12 m.

Step 3: Calculate the surface area.

$$\begin{aligned} SA &= 2B + CH \\ &\doteq 2(50.24) + 25.12(10) \\ &\doteq 100.48 + 251.2 \\ &\doteq 351.68 \\ &\doteq 352 \end{aligned}$$

The surface area is about $352\ \text{m}^2$.

2. a. **Step 1:** Calculate the area of the base.

$$\begin{aligned} B &= \ell w \\ &= 16(7) \\ &= 112 \end{aligned}$$

The area of the base is $112\ \text{mm}^2$.

Step 2: Calculate the perimeter of the base.

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2(16) + 2(7) \\ &= 32 + 14 \\ &= 46 \end{aligned}$$

The perimeter of the base is 46 mm.

Step 3: Calculate the surface area of the prism.

$$\begin{aligned} SA &= 2B + PH \\ &= 2(112) + 46(5) \\ &= 224 + 230 \\ &= 454 \end{aligned}$$

The surface area is $454\ \text{mm}^2$.

b. **Step 1:** Calculate the area of the base.

$$\begin{aligned} B &= \frac{nsa}{2} \\ &= \frac{5(8)(5)}{2} \\ &= \frac{200}{2} \\ &= 100 \end{aligned}$$

The area of the base is $100\ \text{cm}^2$.

Step 2: Calculate the perimeter of the base.

$$\begin{aligned} P &= ns \\ &= 5(8) \\ &= 40 \end{aligned}$$

The perimeter of the base is 40 cm.

Step 3: Calculate the surface area of the prism.

$$\begin{aligned} SA &= 2B + PH \\ &= 2(100) + 40(35) \\ &= 200 + 1400 \\ &= 1600 \end{aligned}$$

The surface area is $1600\ \text{cm}^2$.

c. **Step 1:** Find the area of the base.

$$B = \frac{bh}{2}$$
$$= \frac{12(10)}{2}$$
$$= \frac{120}{2}$$
$$= 60$$

The area of the base is $60\ \text{m}^2$.

Step 2: Calculate the perimeter of the base. **Note:** The sides of the triangle are 12 m, 20 m, and 24 m.

$$P = s_1 + s_2 + s_3$$
$$= 12 + 20 + 24$$
$$= 56$$

The perimeter of the base is 56 m.

Step 3: Calculate the surface area of the prism.

$$SA = 2B + PH$$
$$= 2(60) + 56(15)$$
$$= 120 + 840$$
$$= 960$$

The surface area of the triangular prism is $960\ \text{m}^2$.

d. **Step 1:** Calculate the area of the base.

$$B = \frac{nsa}{2}$$
$$= \frac{6(20)(17)}{2}$$
$$= \frac{2040}{2}$$
$$= 1020$$

The area of the base is $1020\ \text{cm}^2$.

Step 2: Calculate the perimeter of the base.

$$P = ns$$
$$= 6(20)$$
$$= 120$$

The perimeter of the base is 120 cm.

Step 3: Calculate the surface area of the prism.

$$SA = 2B + PH$$
$$= 2(1020) + 120(30)$$
$$= 2040 + 3600$$
$$= 5640$$

The surface area of the prism is $5640\ \text{cm}^2$.

Enrichment

1. **a.** $3\ m^3 = 3\,000\,000\ cm^3$ **b.** $7800\ mm^3 = 7.8\ cm^3$

c. $1.82\ m^3 = 1\,820\,000\ cm^3$ **d.** $450\,000\ mm^3 = 450\ cm^3$

2. **a.** $0.5\ km^3 = 500\,000\,000\ m^3$

b. $75\,000\,000\ cm^3 = 75\ m^3$

c. $1.2\ km^3 = 1\,200\,000\,000\ m^3$

d. $592\,000\ mm^3 = 0.000\,592\ m^3$

3. **a.** $B = \pi r^2$ $V = BH$

$\doteq 3.14\left(51^2\right)$ $\doteq 8167.14(137)$

$\doteq 3.14(2601)$ $\doteq 1118898.18$

$\doteq 8167.14$

$d = 102$ cm
$r = 51$ cm

The volume of the roll of newsprint is about $1\,118\,898.18\ cm^3$.

b.

fewer larger units — more smaller units

km^3	hm^3	dam^3	m^3	dm^3	cm^3	mm^3

$\div(1000\times1000)$

To change from cubic centimetres to cubic metres divide by 1 000 000.

$1\,118\,898.18\ cm^3 \doteq 1.118\,898\,18$

$\doteq 1.12\ m^3$

The volume of the roll of newsprint is about $1.12\ m^3$.

4. **a.** $B = \ell w$ $V = BH$

$= 90\times80$ $= 7200(75)$

$= 7200$ $= 540\,000$

The volume of the jewellery box is $540\,000\ mm^3$.

b.

fewer larger units — more smaller units

km^3	hm^3	dam^3	m^3	dm^3	cm^3	mm^3

$\div1000$

To change from cubic millimetres to cubic centimetres divide by 1000.

$540\,000\ mm^3 = 540\ cm^3$

The volume of the jewellery box is $540\ cm^3$.

5. $50\,000\times0.08 = 4000$

The volume of the ice is $4000\ m^3$.

$5\ km^2 = 50\,000\ m^2$
$8\ cm = 0.08\ m$

Coxeter Groups

Date: 1950s

Inventor: Harold Scott MacDonald (Donald) Coxeter, mathematician, born in London, England, in 1907.

Significance: As well as being significant purely as an elegant expression of abstract mathematical truth, the insights summarized in what are known as Coxeter groups have found practical application in helping crystallographers understand how carbon atoms are arranged in a diamond.

Profile: When he was just three years old, Harold Scott MacDonald Coxeter used to look at the stock market reports in the newspaper, fascinated by the columns of numbers. Later he became interested in shapes as well, in pyramids and cones and other three-dimensional solids. Before he was 16 he was contemplating "the analogues of regular solids in four and more dimensions;" his prize-winning essay on the subject brought him to the attention of Bertrand Russell and other mathematical luminaries. After abandoning the idea of a musical career—two well-known musicians had made discouraging assessments of his compositions—Coxeter decided, instead, to go in for mathematics. He won a scholarship to Cambridge University, completed his doctorate there in geometry and came to the University of Toronto where, since 1935, this vegetarian and pacifist has been teaching, generously sharing his ideas, doing research and winning renown in the world of ideas.

Geometry, according to Coxeter, is the study of shapes and patterns. A century ago, when the subject was in vogue, many entirely unexpected properties were discovered in elementary geometrical objects such as triangles and circles. But when Coxeter began his career, geometry had become passé. Indifferent to intellectual fashion, he became one if its consummate practitioners and, by his many contributions, helped bring about the revival now underway.

Coxeter has cracked open, clarified, generalized, enumerated, classified and solved a multitude of geometrical problems. He has studied symmetry in such complex and unreal realms as that of the polytopes, abstract generalizations into higher dimensions of two-dimensional polygons (such as squares) and three-dimensional polyhedra (such as cubes). And his systematic formulation of the "grammar" of symmetry in multi-dimensional kaleidoscopes, a set of dizzily abstract concepts neatly organized and simply expressed in terms of algebra and diagrams, are what are now known as "Coxeter groups."[1]

[1] Sean McCutcheon, "Discoveries and Inventions," *Horizon Canada*, vol. 9, no. 101 (1987): third cover.

Offshore Drilling Platforms of Floating Ice

Inventor: Dr. Hans Kivisild, engineer, born in Estonia in 1922, now vice president of Lavalin Offshore Inc. in Calgary.

Significance: Drill rigs mounted on floating ice platforms have confirmed the existence of large volumes of natural gas off the shores of Canada's Arctic islands.

Profile: One winter's day in 1950, Hydro-Québec asked an engineering firm to investigate ice flooding in the river flowing around the north shore of the island of Montreal. Senior engineers refused what would be a wet and cold job, so it was assigned to Hans Kivisild who, though he had a doctorate from the Royal University of Technology in Stockholm, was a relatively junior engineer. The key factor controlling ice accumulation, Kivisild's report showed, was not water temperature, as most engineers then held, but the speed at which water was moving. He became an expert on ice, and taught his colleagues to see the substance in new ways.

Looking for Gas in the Arctic: Geologists have always suspected that large pools of oil and natural gas are trapped in the sedimentary rocks of Canada's Arctic. The only way to prove such suspicions is to drill. Major petroleum development was underway in the Canadian Arctic in the 1960s. Exploration drilling began on land, both in the Mackenzie delta and farther north on Melville Island. The geologists' suspicions were confirmed, and it became apparent that discoveries could be made in the rocks of the sea floor adjacent to the delta and the Arctic Islands. The Beaufort Sea being shallow, it was possible to build artificial islands of sand and steel sitting on the ocean floor. But how to build a drilling platform in the deeper waters between the islands of the Arctic archipelago?

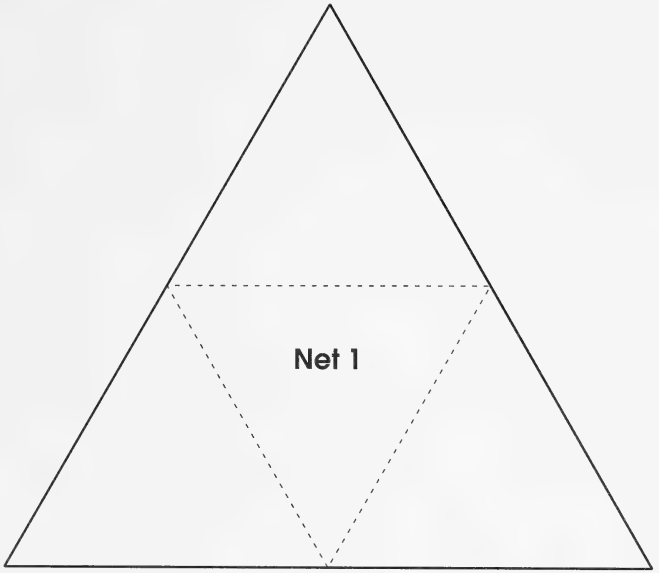
For eight months of the year, sea ice in the archipelago is shorefast; that is to say, a solid sheet of floating ice is locked to the island shores and covers the seas between them. This natural ice cover is, on average, about 2 m thick. Kivisild knew this was not strong enough to support the weight of drilling equipment, but he also knew how ice behaved and in 1969 he suggested building up a thick, strong platform of ice, using layers of sea water frozen by the bitter cold as a construction material.

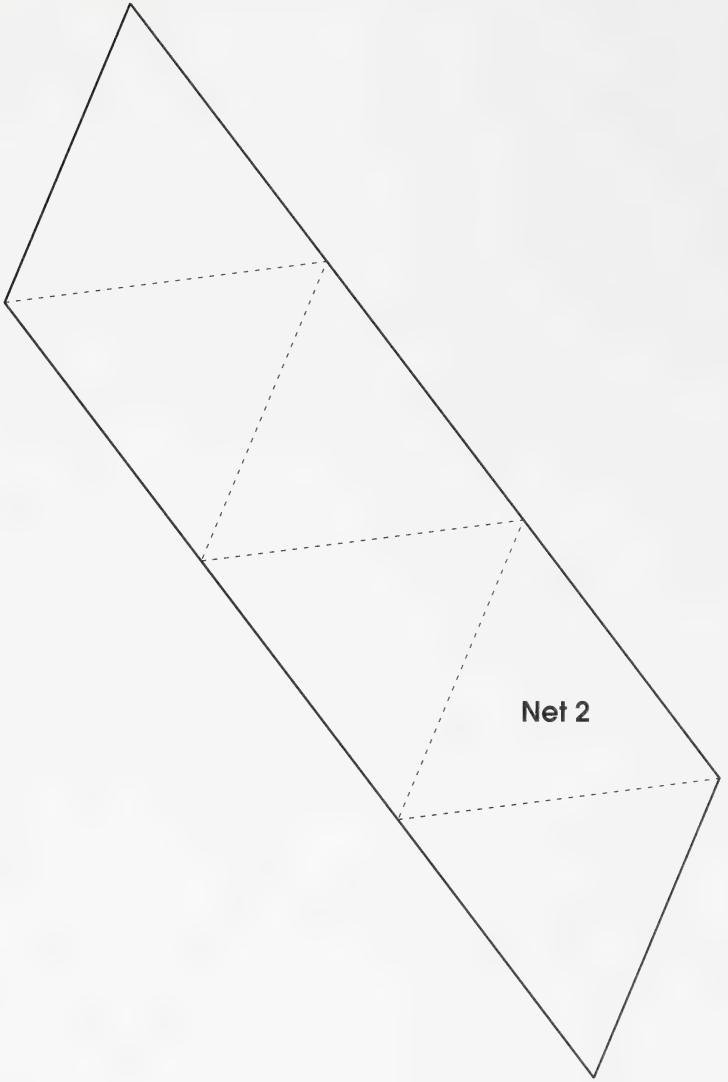
Kivisild's suggestion was adopted in 1970 by Panarctic Oils Ltd, a consortium of small, independent companies and the federal government. In the middle of November 1973, Panarctic crews began pumping sea water onto the ice off the coast of Melville Island. During the subsequent six weeks they laid thin layers of ice, one on top of another, five or so layers every 24 hours, forming a lens 5.5 m thick at the centre. Then they put down a layer of insulation and erected a drill rig. Through a hole—the "moonpool"—cut in the ice, they lowered the drill down to, and into, the ocean floor, spudding in the well known as Hecla N-52. They drilled, just as if they were on board a drill ship, until early spring. Then they abandoned the prototype ice platform, leaving it to drift with the partially melting ice sheet during the brief Arctic summer.

The costs, and risks, of any drilling venture are extremely high, especially in the far North, and more especially so offshore. Kivisild's invention has proven to be practical, and cheaper than any of the alternatives.[1]

[1] Sean McCutcheon, "Discoveries and Inventions," *Horizon Canada*, vol. 5, no. 59 (1986): third cover.

Nets





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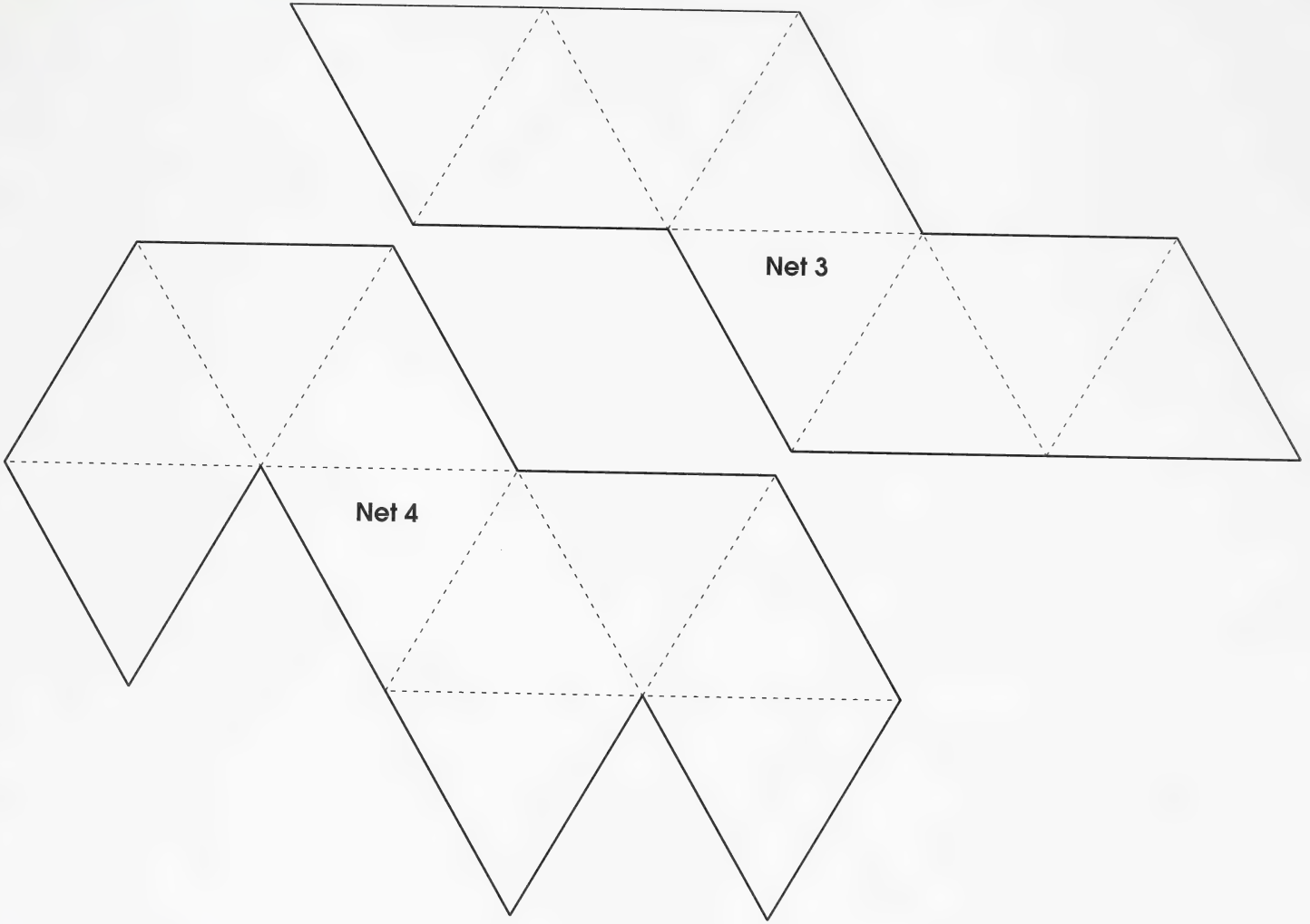

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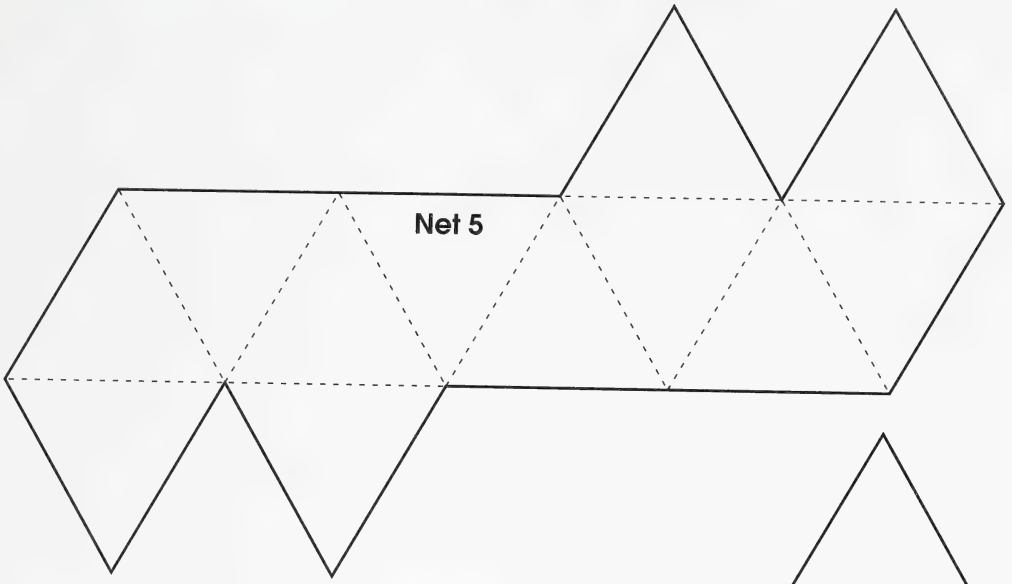

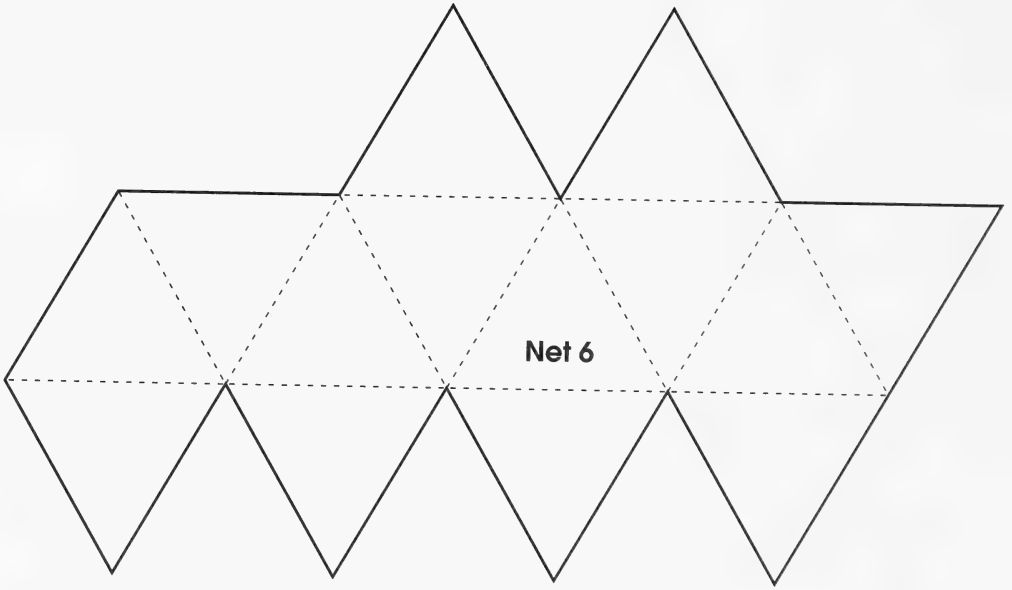

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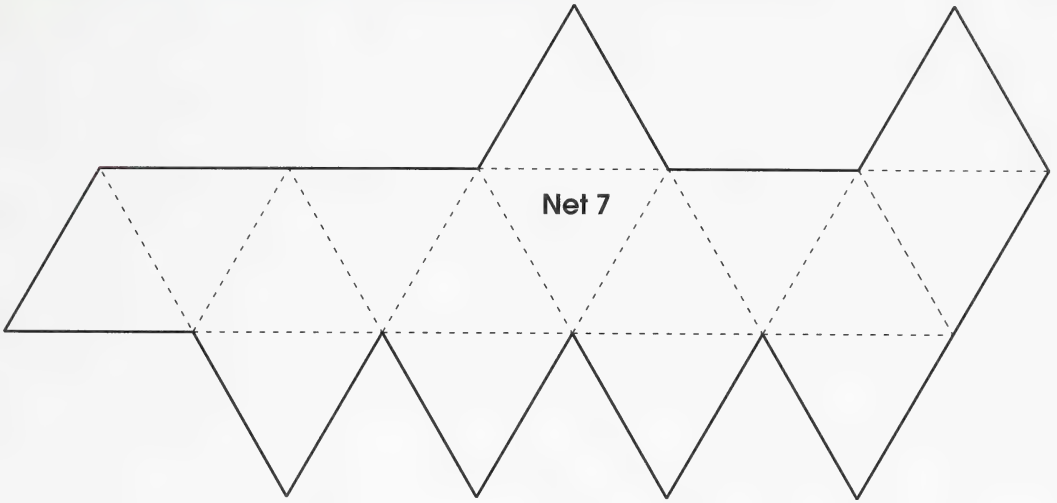

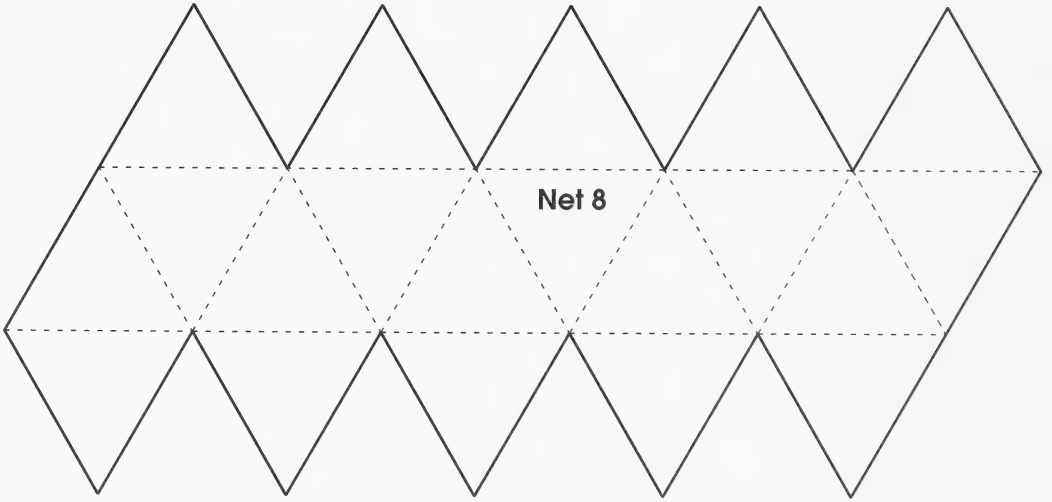

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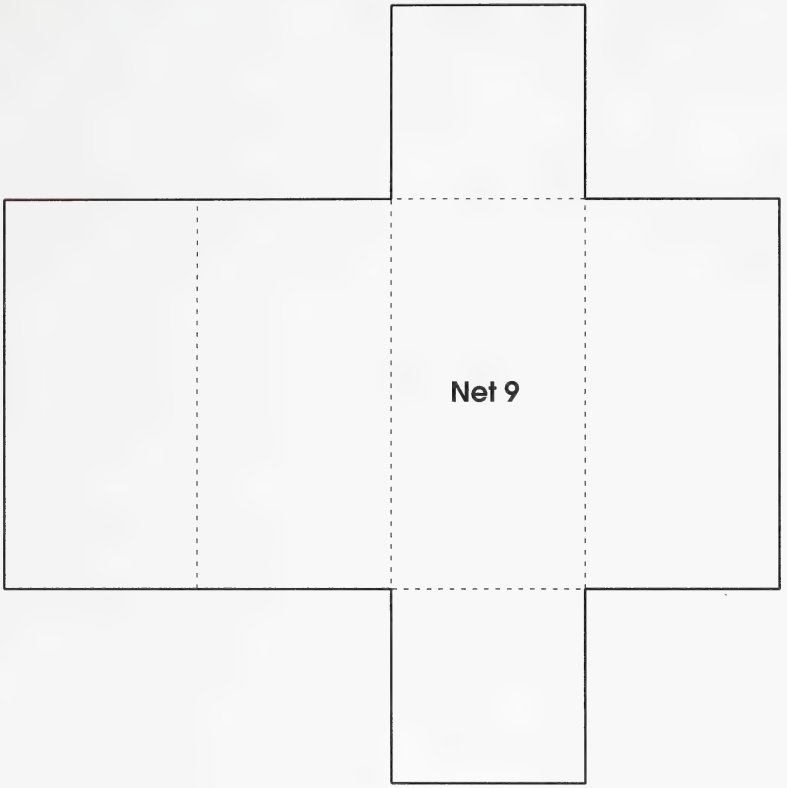

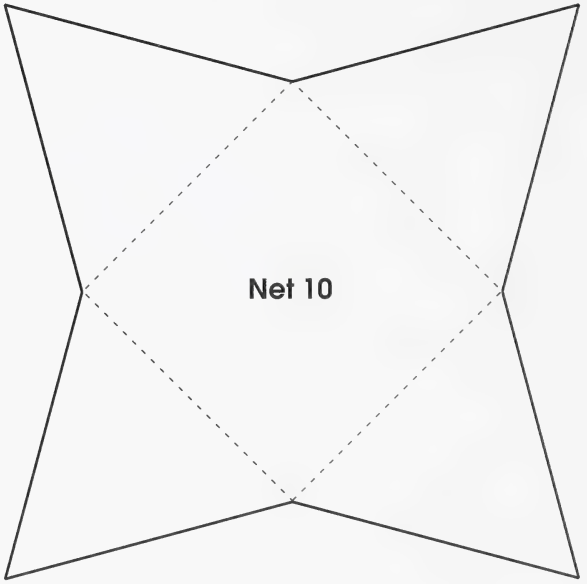

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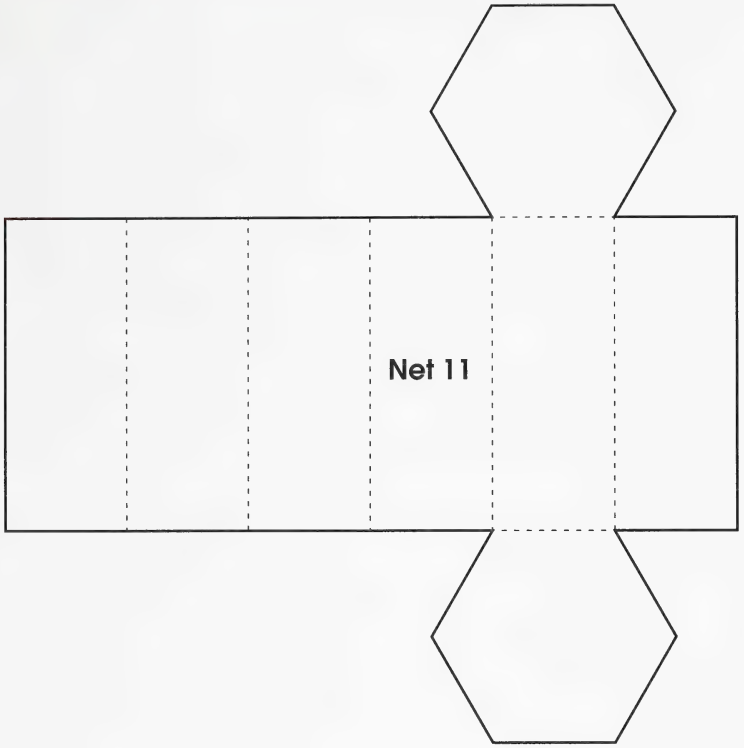

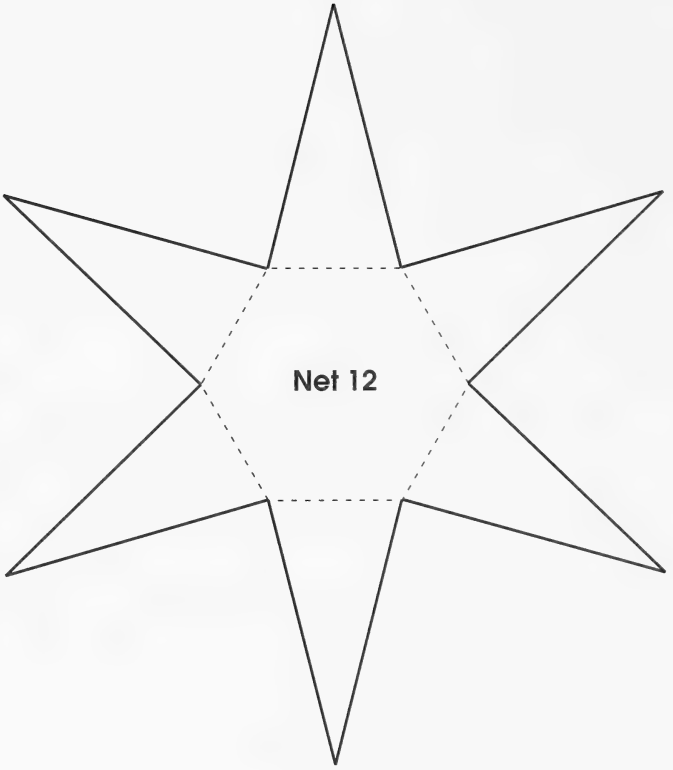

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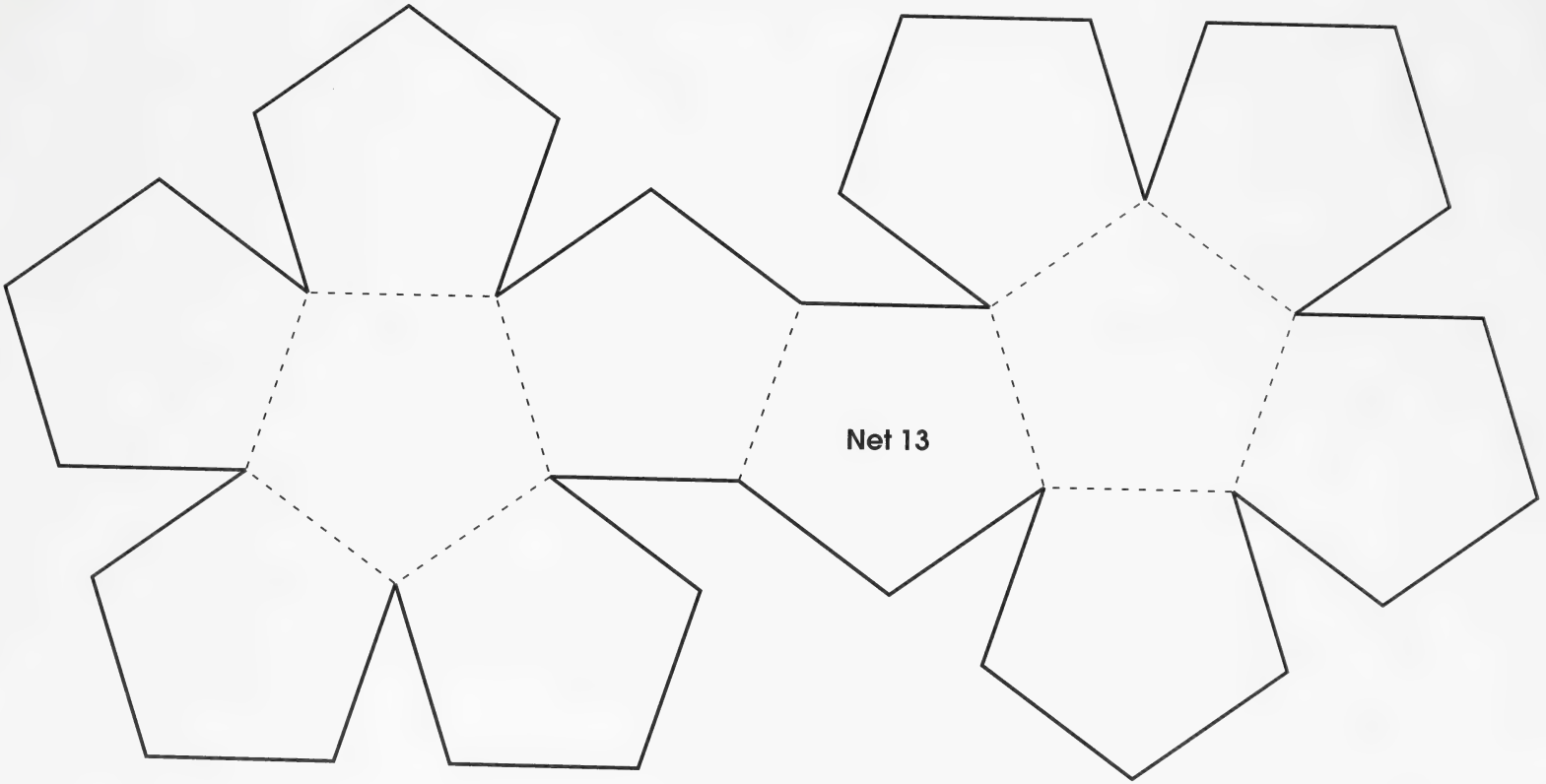

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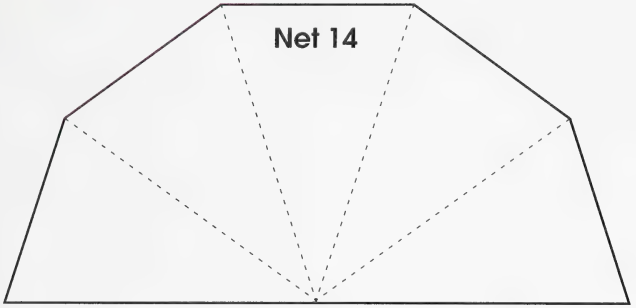

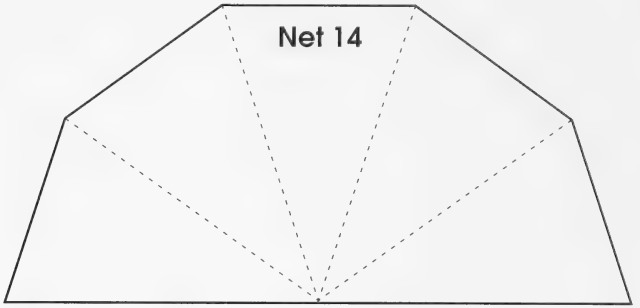

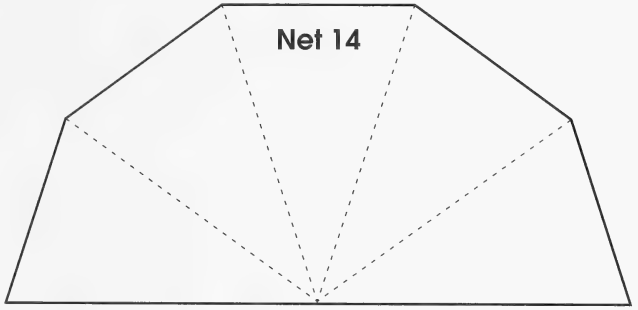

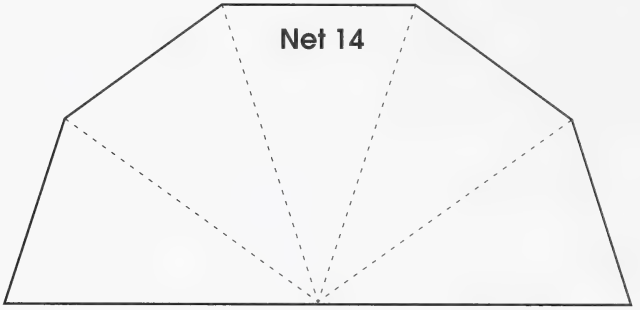

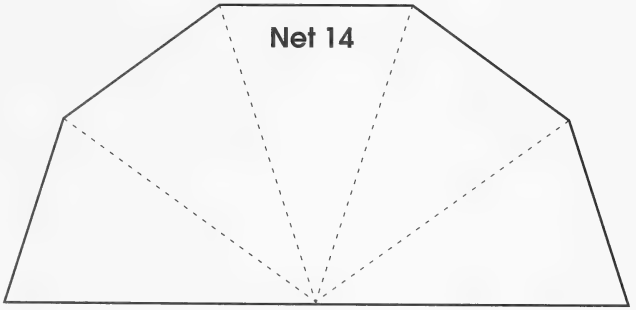

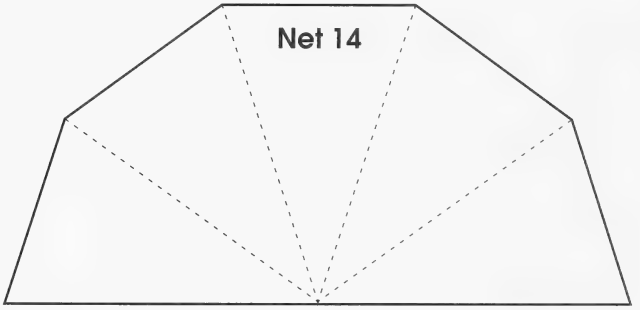

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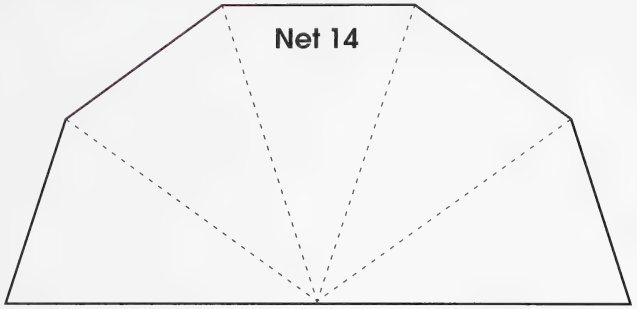
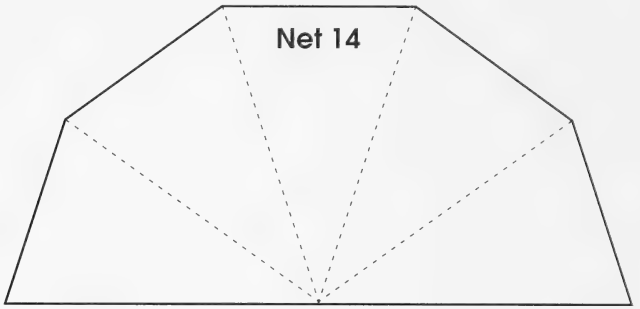
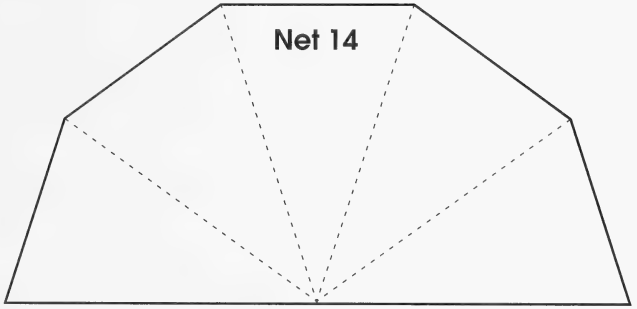
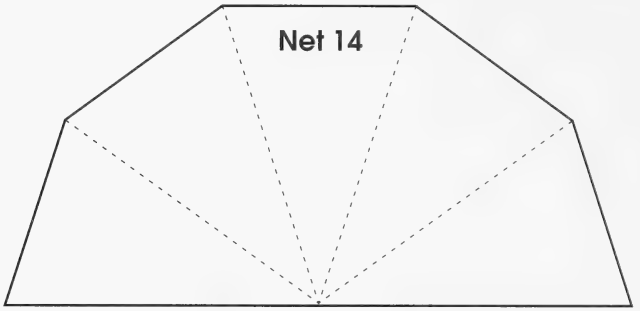
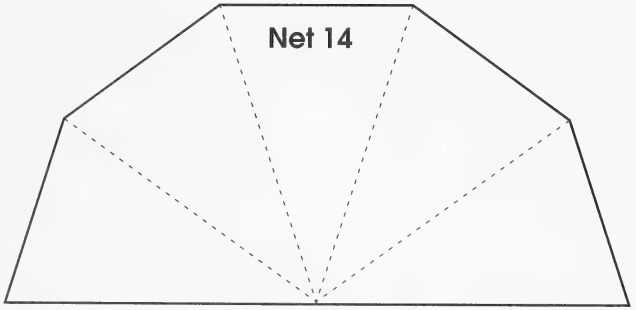
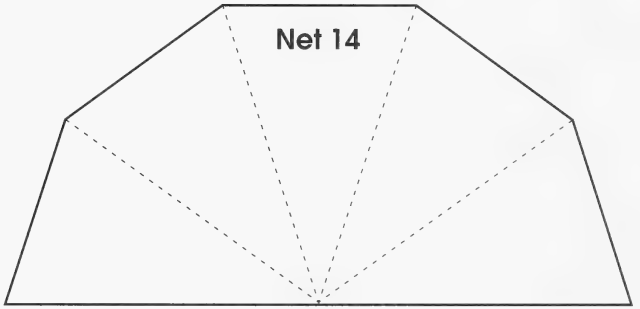




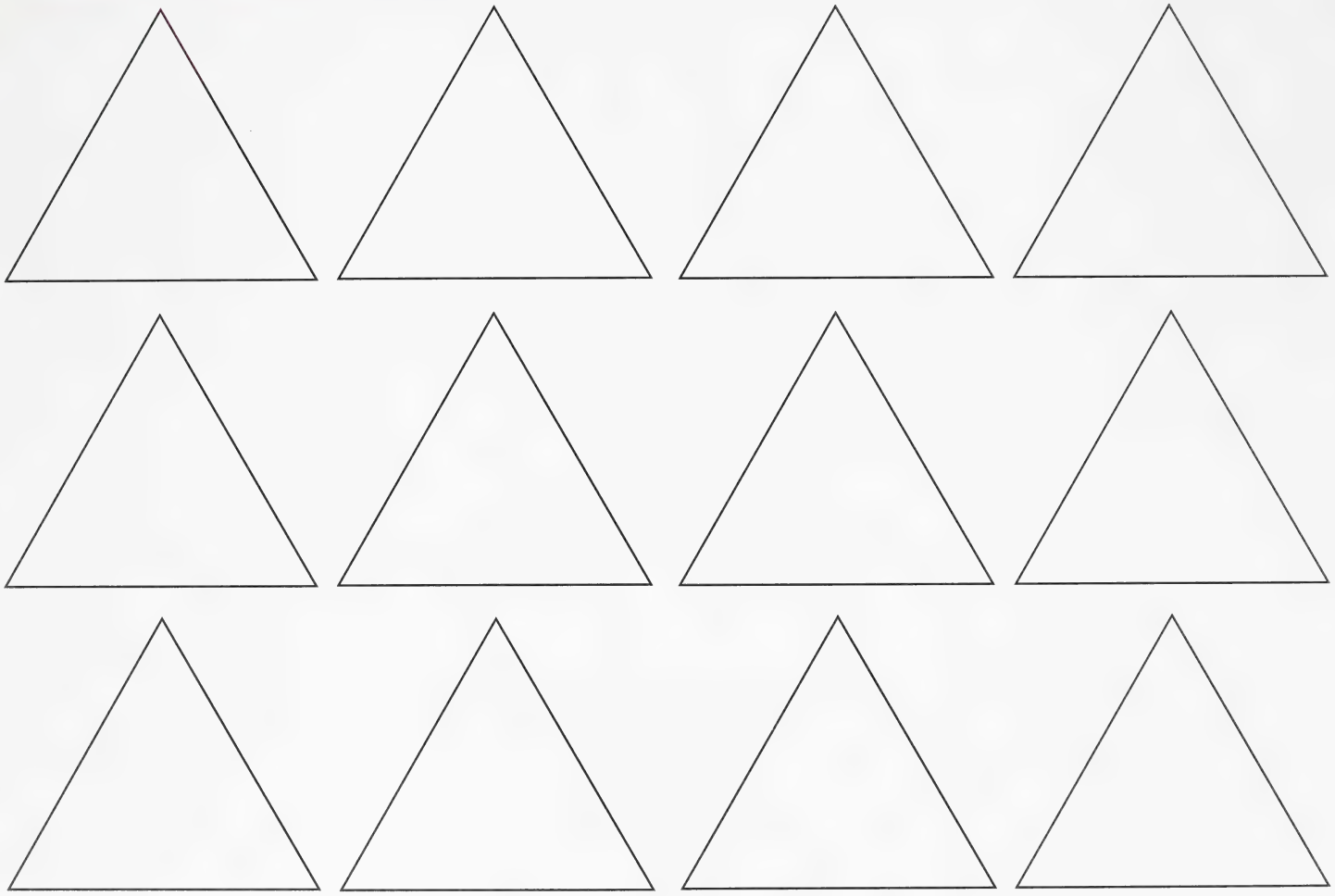








Regular Polygons



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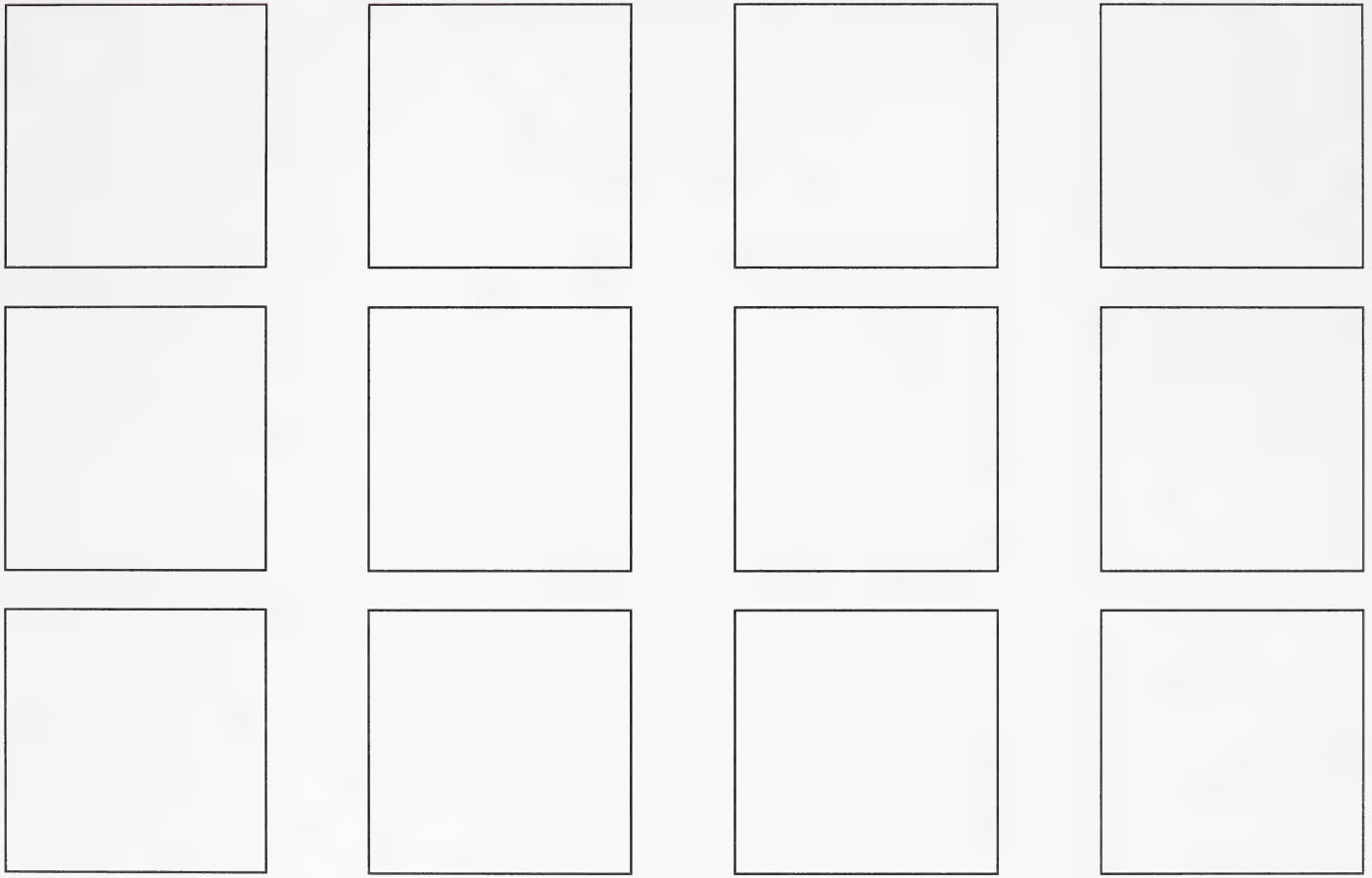
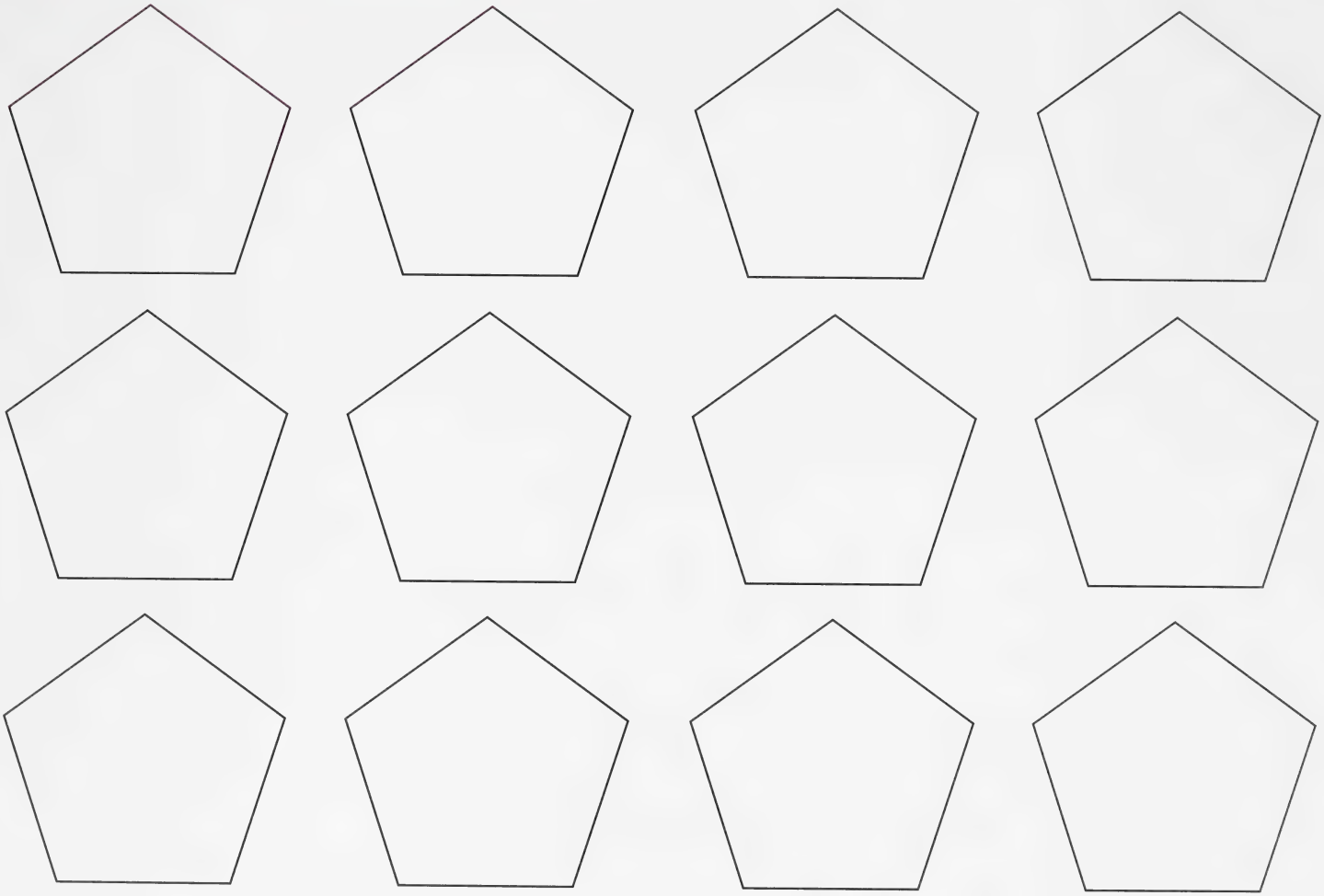


Regular Polygons (continued)



Regular Polygons (continued)

Student and teacher: Use this cover sheet for mailing or faxing.

ASSIGNMENT BOOKLET

8110 Mathematics 8
Module 6

FOR STUDENT USE ONLY

Date Module Submitted:

_____

Time Spent on Module:

_____

(If label is missing or incorrect)

File Number:

_____

Module Number: _____

FOR TEACHER USE ONLY

Assigned Teacher: _____

Module Grading: _____

Graded by: _____

Date Module Received:

_____

Module Assignment Recorded: _____

Student's Questions and Comments

Apply Module Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and module.

Teacher's Comments

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Teacher

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- Has your work been reread to ensure accuracy in spelling and details?
- Is the booklet cover filled out and the correct module label attached?

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MATHEMATICS 8
MODULE 6

Three-Dimensional Geometry

ASSIGNMENT BOOKLET

Distance Learning

Alberta EDUCATION

FOR TEACHER'S USE ONLY

Summary

	Total Possible Marks	Your Mark
Section 1 Assignment	40	
Section 2 Assignment	45	
Final Module Assignment	15	
	100	

Teacher's Comments

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

Mathematics 8
Assignment Booklet
Module 6
Three-Dimensional Geometry
Learning Technologies Branch
ISBN 0-7741-1405-3

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ASSIGNMENT BOOKLET
MATHEMATICS 8 – MODULE 6: THREE-DIMENSIONAL GEOMETRY

Your mark on this module will be determined by how well you do your assignments in this booklet.

Work slowly and carefully. If you are having difficulties, go back and review the appropriate section.

There are two section assignments and one final module assignment in this Assignment Booklet. The total value of these assignments is 100 marks. The value of each assignment is stated in the left margin.

Be sure to proofread each assignment carefully.

40

Section 1 Assignment: Properties of Three-Dimensional Shapes

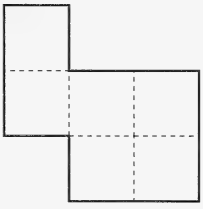
Read all the parts of your assignment carefully and record your answers in the appropriate place.

Circle the letter of the best answer for questions 1 to 4.

(2)

1. Which of the following nets can be used to make a cube?

A.



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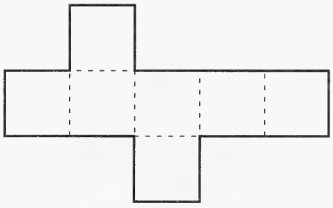

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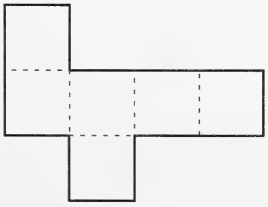

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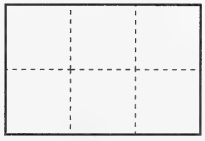
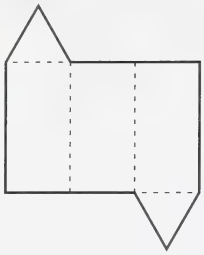


(2) **2.** Which of the following nets can be used to make a triangular prism?

A.



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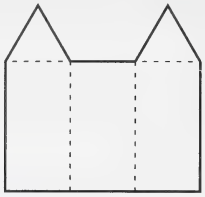

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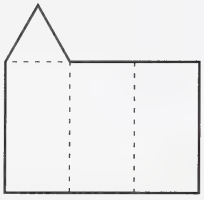

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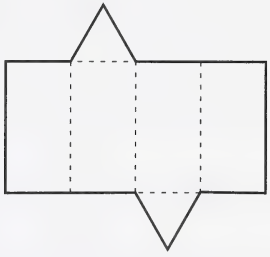
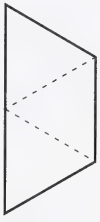


(2) **3.** Which of the following nets can be used to make a triangular pyramid?

A.

B.



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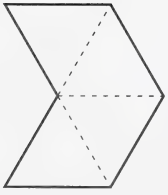
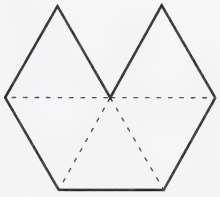
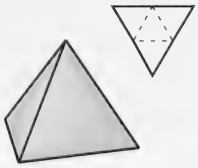


D.

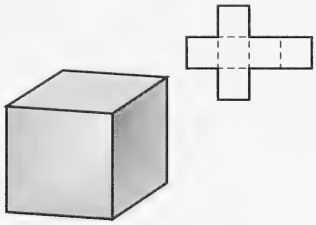


(2) **4.** Following are the sketches of various polyhedra. Beside each is a sketch of the polyhedron's net. The solid of which polyhedron can **not** be used as a die? **Hint:** A die must be completely symmetrical so that each face has an equal opportunity to land face up.

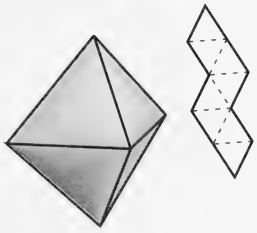
A.



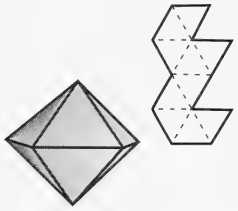
B.



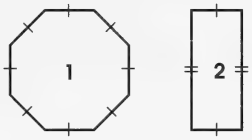
C.



D.

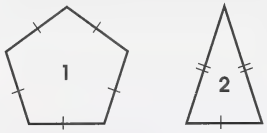


(2) **5. a.** Name the polyhedron that has polygon 1 as a base and polygon 2 as a "side" face.



(2) **b.** Make a sketch of the net of the polyhedron that you identified in question 5.a.

(2) **6. a.** Name the polyhedron that has polygon 1 as a base and polygon 2 as a "side" face.



(2) **b.** Make a sketch of the net of the polyhedron identified in question 6.a.

Clearly show how you arrived at your answers in questions 7, 8, and 9.

(2) **7.** The skeleton of a convex polyhedron is made with toothpicks joined together with mini-marshmallows. If 17 toothpicks and 12 mini-marshmallows are used, how many faces does the polyhedron have?

8. The skeleton of a right prism with a base of 7 sides is made with toothpicks joined together with mini-marshmallows.

(2) **a.** How many toothpicks are needed?

(2) **b.** How many mini-marshmallows are needed?

9. The skeleton of a pyramid is made with toothpicks and mini-marshmallows. The base of the pyramid has 10 sides.

(2) **a.** How many toothpicks are needed?

(2) **b.** How many mini-marshmallows are needed?

(2) **10. a.** If you make a cross section **perpendicular** to the base of a solid cylinder, what shape is the cross section?

(2) **b.** If you make a cross section **parallel** to the base of a solid cylinder, what shape is the cross section?

(2) **11. a.** If you make a cross section **perpendicular** to the base of a solid square pyramid, what shape is the cross section?

(2) **b.** If you make a cross section **parallel** to the base of a square pyramid, what shape is the cross section?

(2) **12.** What do you notice about the cross sections of a solid sphere?

_____

_____

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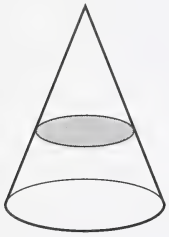
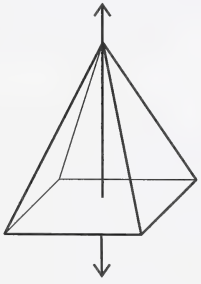
(2) **13.** Is this shaded plane a plane of symmetry for a cone? Explain why or why not.



(2) **14.** Is this line an axis of symmetry for a square pyramid? Explain why or why not.



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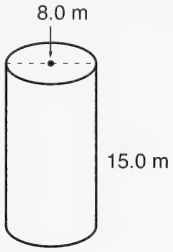
Section 2 Assignment: Measurement of Three-Dimensional Shapes

Read all the parts of your assignment carefully and record your answers in the appropriate place. Clearly show how you arrived at your answers.

1. For each of the following solids calculate the volume and the surface area. **Note:** Round each answer to the nearest tenth.

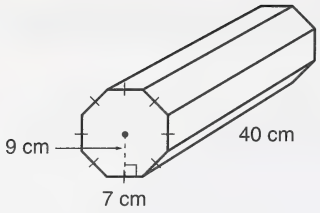
6

a.



(6)

b.



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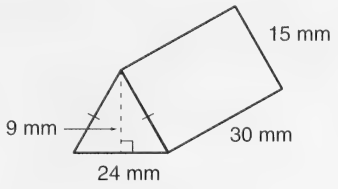
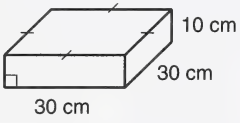
c.



2. Mrs. Smith baked a cake and iced the sides and top of the cake with frosting. The base of the cake is a square $30\ \text{cm} \times 30\ \text{cm}$; the cake is 10 cm high.

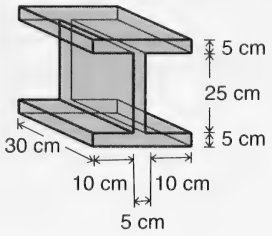
(3) **a.** What is the volume of the cake?

(4) **b.** What is the surface area of the iced portion of the cake?

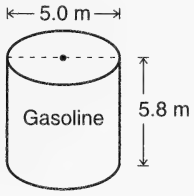


(3) **c.** How can Mrs. Smith cut the cake into 8 pieces so that each person gets the same amount of cake and the same amount of frosting? Make a sketch.

(5) **3.** A concrete block is shaped like this. What is its volume?

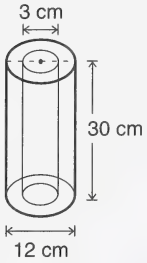


(5) **4.** A painter paints the curved sides and flat top of this gasoline storage tank. If 1 L of paint covers 12 m^2, how much paint is required to cover the surface of the storage tank? **Hint:** Assume there is no wastage and you must purchase whole litres of paint.



(5)

5. a. Paper towelling is wrapped around a hollow cardboard cylinder. In the given diagram, what is the volume of the paper towelling? **Note:** Round to the nearest tenth. (The diagram is not drawn to scale.)



(2)

b. Why do you think some manufacturers of paper towelling use a cardboard cylinder with a wider diameter than other towel rolls?

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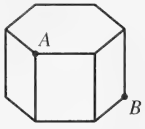
Final Module Assignment

Read all the parts of your assignment carefully and record your answers in the appropriate place. When answering the following questions, be sure to clearly show how you arrived at your answers.

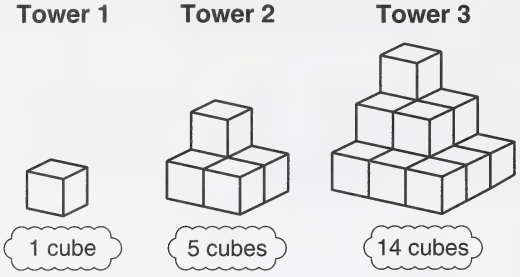
4

1. A spider wants to crawl from point A on the outside of an aquarium to point B on the outside. The aquarium is a hexagonal prism. Each side of the hexagonal base is 30 cm and the apothem is 25 cm. The height of the aquarium is 80 cm.



What is the length of the shortest path the spider can crawl to get from point A to point B?

2. Cubes that are 1 cm × 1 cm × 1 cm are stacked in square layers to form a pattern of towers as shown.



Hint: You may use sugar cubes, base ten blocks, or other small blocks to help you answer these questions.

Suppose you painted the exposed surfaces of each tower.

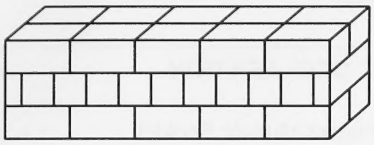
(2) **a.** What is the area of the painted surfaces in the second tower?

(2) **b.** What is the area of the painted surfaces in the third tower?

(2) **c.** Draw a sketch of the fourth tower in the pattern.

(2) **d.** What is the area of the painted surfaces in the fourth tower?

(3) **3.** Bricks 8 cm × 10 cm × 20 cm are used to build this wall. What is the volume of the wall?



ASSIGNMENT BOOKLET DECLARATIONS

The Student's Declaration is to be filled in by a student registered at the Alberta Distance Learning Centre. If the student is under 16, the Learning Facilitator's Declaration is to be filled in by the learning facilitator. Failure to complete this page may invalidate the assignment results.

STUDENT'S DECLARATION

- I have followed the instructions outlined in the Student Module Booklet.
- I have completed the activities to prepare myself for the assignments in this Assignment Booklet.
- I completed the assignments in this Assignment Booklet by myself.

_____

Student's Signature

LEARNING FACILITATOR'S DECLARATION

I hereby certify that I have supervised the learning activities completed by _____.

Student's Name

I also certify that to the best of my knowledge the assignments in this Assignment Booklet were completed independently by this student.

_____

Supervisor's Signature

If you, the student or learning facilitator, have any comments or observations regarding this module, write them in the following space.

_____

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